

A Guide to the IMF's 16th Quota Review

G-24 Working Paper

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A Guide to the IMF's 16th Quota Review

TOMÁS GONZÁLEZ^{*†‡}

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Abstract

The quota review is arguably the most important discussion in the IMF. The purpose of this paper is to provide a structured guide to the quota formula and quota review process in light of the most recent data—which shows a substantial misalignment between current quotas and the relative importance of Emerging Market and Developing Countries in the world economy. The analysis presented is based on a model built to replicate past reform proposals that taught us the simplicity of the formula can be deceptive. The paper can be roughly divided in two parts: the first introduces the quota review, the roles of quotas and key formula properties that underlie the main results; the second explores the likelihood of an agreement. Voting simulations show the formula alone will be insufficient for consensus given how changes in its components affect differently various IMF members. Any agreement will probably require external elements to convince an important part of the membership to reduce their quotas. Results also show there is sufficient scope for groups of countries to coalesce around reforms that protect their most important interests, providing a path to simplify the negotiation process. In the end changes to the quota formula alone will not secure an agreement, but the formula will likely continue being the organizing and legitimacy mechanism for quota reviews.

I. QUOTAS AND THE QUOTA REVIEW

THE quota review is perhaps the most important discussion in the IMF. On the surface it may appear as a simple financing exercise, but it really is a much more complex process that brings together the political and technical sides of the Fund's work, and forces members to reexamine the institution's goals and priorities and to reconcile them with their own. And it is a process that should lead to decisions that ensure the Fund is adequately resourced to fulfill its current and future obligations.

Quotas have been at the center of the IMF's governance since its creation at the Bretton Woods Monetary and Financial Conference in 1944. The original Articles of Agreement determined that each member would be assigned a quota expressed in special drawing rights (SDR) and established its level for each of the 44 founding members. They also stipulated that the subscription of each

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member would be equal to its quota and contained provisions for its adjustment, indicating that "the Board of Governors shall at intervals of not more than five years conduct a general review and, if it deems appropriate, propose an adjustment of the quotas of members"¹. Their goal was to ensure quotas adequately reflected the world's economic and political post-war order.

The role of quotas has solidified since and continues to encompass all the fundamental interactions between members and the Fund. Today they play a critical role in four key areas that form the backbone of the quota review discussions.

- **Fund's size and policies.** Quotas are the IMF's main source of resources, providing a significant portion of the SDR 780 billion (roughly 1 USD trillion) of current lending capacity. In the wake of the Global financial crisis quota resources had to be supplemented with bilateral borrowing agreements (BBA)—loans and note repurchasing agreements from the official sector—and the new agreements to borrow (NAB)—agreements in which a group of advanced and developing economies stand ready to lend to the Fund pre-agreed amounts. This borrowing allowed lending capacity to reach 4.1 times the pre-crisis level, a substantial increase in resources to deal with future crises. The problem, however, is they are temporary. SDR 230 billion of bilateral agreements were set to expire at the end of 2020 and NAB in November 2022. In January 2022 the Fund announced it had successfully secured a doubling of the NAB and a new round of BBAs to maintain its lending capacity at around USD 1 trillion for the coming years².

Staff generally use three methodologies to assess the adequacy of resources relative to potential demand. The first looks at the *levels of quotas relative to a set of economic indicators* like GDP or external financing needs to determine resources required to match a weighted average of the selected historical ratios. The second is an *access-based approach* that combines program sizes for past top borrowers that simultaneously sought help from the Fund, and a panel logit to estimate the demand for Fund's resources in response to a global volatility shock. Finally, a *global scenarios approach* applies a shock with an intensity between the 65th to 95th percentile of past crises to the countries deemed to have the largest vulnerabilities according to the IMF Vulnerability Exercise..

These methodologies usually produce a broad range of estimates leading to very different discussions in terms of what member countries need to do to keep the Fund adequately resourced. A country can find itself having to decide between doing very little at the lower end of the spectrum, to committing an amount equivalent to several times its quota in the upper end. Qualitatively these are very different conversations that complicate internal consensus. To provide guidance staff have argued that a new crisis with a magnitude similar to the average crisis since the 1980s would imply financing needs at least similar to the Fund's current lending capacity.

In any case the final decision on a quota increase will have important implications for instruments and policies. A significantly smaller IMF would have to be more restrictive in giving access to resources—implying program characteristics would probably need to change and, perhaps, certain types of programs would have to be abolished altogether. This could not only strain the relationship with member countries that see their access reduced,

¹See Article III, sections 1 and 2.

²See [International Monetary Fund \(2022\)](#) and for details.

but also lead to a smaller Fund that loses influence if it is perceived to have less capacity. to respond to liquidity needs The opposite could also be the case, if a Fund that is seen as too large could incentivize moral hazard behavior by relaxing access criteria to ensure full use of an expanded lending capacity.

- **Voting power.** Quotas almost fully determine a member's voting power. This is so because the total number of votes a country has is equal to the sum of its basic votes plus 1 vote for each SDR 100,000 of quota. In the case of basic votes, they were introduced in the original articles of agreement and set equal to 250 for each founding member³. This remained unchanged until the quota reform of 2008 in which they were tripled to increase the voting share of low-income countries. The reform fixed the share of basic votes to total voting power at the current 5.502 percent so that each member would have 750 basic votes⁴.

Regarding quotas, they were originally determined by the negotiation that led to the Articles of Agreement. For the United States it was set at US\$ 2,750 million, and at US\$ 0.5 million for the smallest members Liberia and Panama. The rest of the founding members were allocated quotas between these two, with the UK having roughly half the quota of the US and the Soviet Union US\$ 1,200 million. Germany and Japan were not part of the original agreement and China was allocated a quota of US\$ 550 million. Today, after several increases, actual quota shares (AQS) stand at the levels indicated by figure 6 with the largest country having 17.4 percent (US) and the smallest 0.00052 percent (Tuvalu).

It is important to note that since quotas are calculated relative to economic weight, the bias towards the weakest members introduced by basic votes is more than compensated, leading to an almost perfect correlation between quotas and voting power. Note also that although most decisions require a simple majority, special decisions like the adjustment of quotas require special voting majority of 85 percent⁵. This gives the United States an effective veto power as it is the only member currently holding more than 15 percent of total votes.

- **Program access.** Quotas determine to a large extent the access members can have to Fund resources. This is the result of access limits set both by the Articles of Agreement and the Executive Board, and are expressed as a percentage of a member's quota. An example are Stand-By Arrangements, the Fund's most common type of instrument, that have an annual access limit equal to 145 percent of quota and a cumulative limit of 435 percent of quota. All current credit facilities have a similar structure, with period-long and cumulative limits based on quota levels. The only exception is the Flexible Credit Line—designed to address balance of payments needs for countries with very strong fundamentals—that currently has no access limits⁶.

³Article XII, Section 5

⁴Since basic votes are determined dividing 5.502 percent of the total voting power equally among members, the number of basic votes will vary with changes in quota sizes or the number of IMF members. The adjustment to any of the two is automatic by applying the percentage to the new total number of votes and rounded to avoid fractional votes. See [International Monetary Fund \(2008\)](#) for details. This reform became effective in 2011.

⁵See [Articles of Agreement](#) and [International Monetary Fund \(2018e\)](#) for details.

⁶There is however an exceptional access policy that can allow access beyond normal limits. The policy was introduced in 2002 to give greater clarity on the circumstances that would trigger a special response by the Fund and on the conditions required for such access. In December 2021, for example, the Fund approved 18-month extensions to the cumulative access limits under its emergency financing instruments and allowed all other access limits that had been temporarily increased to return to their pre-pandemic levels.

This policy on access limits was designed to meet several objectives. Access levels should ensure balance of payment needs are adequately met, give members an ex-ante indication of the magnitude of the available financing so they approach the Fund at the appropriate time when needs can be addressed within the specified limits, and act as a risk management tool by avoiding excessive loan concentration. Setting limits in terms of quotas also aims to ensure an evenhanded treatment of countries⁷

The pricing structure for all instruments also includes an element of size and an element of duration so that there is an increase in the cost of a loan when it surpasses a specific quota level or length of time. The pricing of the Stand By Arrangement, for example, includes a surcharge of 200 basis points for amounts above 187.5 percent of quota, plus an additional 100 basis points when the outstanding credit remains above that level for more than 36 months⁸

- **Subscriptions and allocations.** Quota subscriptions determine the size of the resources a member country must pay the IMF. They form the basis from which lending operations are financed and must therefore be paid in full—with at least a quarter in SDRs or a foreign currency acceptable to the Fund. Allocations, on the other hand, are the mechanism under the Articles of Agreement by which the IMF can supplement existing international reserve assets to create liquidity. They are an unconditional allocation of SDRs to member countries in proportion to their quotas in which countries do not need to meet any special conditions to receive them, and where once in their possession no restrictions are placed on how they can be used to address balance of payments needs.

Allocations are not as common as subscriptions, with only four having been made to date. The first two were in 1970–72 and 1979–81, for a combined value of 12.1 billion. The third, an SDR 162 billion allocation that took place in the second half of 2009 to help address the global need for liquidity during the global financial crises. It was designed to partially alleviate the need of the worst hit countries to rely on contractionary policies for adjustment. It was followed by a special allocation of SDR 21.5 billion to correct historical problems and ensure all members could participate in the SDR system⁹. The last one was a general allocation equivalent to USD 650 billion that was completed in August 2021 aimed at improving resilience and stability in the wake of the COVID pandemic¹⁰

By having to deal with these multiple issues, quota reviews go well beyond assessing financial sufficiency. Logistically, though, they are a three-part process that simultaneously (i) determines to what extent existing quotas are capable of meeting members' needs; (ii) if more resources are needed decides how to split them between borrowing and quotas; and (iii) distributes any quota increases among member countries. And reviews must be carried out at 5-year intervals and need to secure the minimum 85 percent support for approval.

There have been 15 reviews so far, with the 16th currently underway, which is scheduled to be completed by the end of 2023. Unsurprisingly, seven of the reviews have ended with no increase in quotas, the average increase for the others has been 56.7 percent, and in five the increase has been less than 50 percent. Only one—completed in 2010 in the aftermath of the global financial

⁷See [International Monetary Fund \(2018e\)](#) for details.

⁸IMF Factsheet for the Stand-By Arrangement (2021).

⁹See Articles XV and XVIII and [International Monetary Fund \(2018e\)](#) for details.

¹⁰See [International Monetary Fund \(2021a\)](#) for details.

crisis—reached a 100 percent overall increase.

Finally, quota increases can be done in one of three ways: *equiproportional* in which the increase is distributed to all members in proportion to existing quotas and hence without altering the existing distribution; *selective* in which the increase is allocated in accordance with the calculated quota shares; and *ad-hoc* where increases are based on a specific agreed criteria usually designed to address country-specific problems related to underrepresentation. Historically all but one of the increases have included an equiproportional element, six an ad-hoc element and only four a selective component. In fact, the last review was atypical in the sense that it was the first not to include an equiproportional element and the only one to have had a selective element greater than the ad-hoc component.

II. THE CURRENT FORMULA

So far we have focused on the review process and adequateness of Fund's resources, without reference to how quotas for individual countries are determined. Quotas have always sought to reflect the relative importance of members in the world economy. Originally, Bretton Woods devised a formula in which a country's quota was defined as the linear combination of national income, gold and foreign exchange reserves, annual imports, and the maximum variability in exports; all multiplied by a factor of 1 plus the share of exports in national income¹¹.

Variables were measured over different periods of time (e.g. income at its 1940 level, while trade variables were averaged over 1934–38 period) and distinct weights assigned, with variability having twice the weight of reserves and five times the weight of income. Interestingly, the formula had to accommodate a series of additional constraints like a predetermined size for the Fund, the fact that the US quota had to be roughly twice the size of the UK quota or the fact that, in descending order, the largest quotas had to go to the US, the UK, the USSR and China. This formula reflected both the economic thinking and political feasibility at the time, and paved the way for how future reforms would approach formula changes and quota reviews.

Three formula reforms have been made since. The first in 1962–63, where the single formula was replaced by a multiple-formula approach designed to raise the quotas of members with small open economies. The reform changed the coefficients of the original formula and redefined variability to ensure it aligned better with the Fund's instruments. Under this approach five formulas were applied to two data sets and ten variables calculated to construct a quota range for each member, with the final quota chosen to be the value at the higher end of the range. An additional reform was made in 1983 that replaced national income with GDP, reduced the weight of variability and modified the way in which the quota range was calculated.

The last reform took place in 2008 as part of the wider Quota and Voice Reforms introduced by the 14th review¹². The aim was to reduce the complexity of the multi-formula approach and increase transparency, while ensuring consistency with the different roles of quotas. This reform

¹¹See [International Monetary Fund \(2000\)](#) for details.

¹²In addition to formula changes, the 2008 reforms included a doubling of quotas to increase resources available to support member's needs, raise the quota share of dynamic developing economies and protect the poorest members by trebling basic votes. In total, the reform increased the voting share of a third of the membership, and a 4.1 percent shift in quotas for the most dynamic emerging market countries

produced the formula in use today, in which the quota of an individual country i is given by

$$Q_i = (0.5 y_i + 0.3 o_i + 0.15 v_i + 0.05 r_i)^k, \quad (1)$$

where¹³

- The **calculated quota share** (Q_i) is expressed as a linear combination of the variables weighted to capture economic importance and potential need for resources;
- **GDP** (y_i) is a blend of GDP converted at market rates and purchasing-power-parity exchange rates—with weights of 0.60 and 0.40, respectively—averaged over a three-year period and expressed as a share of the sum for all countries;
- **Openness** (o_i) is the annual average of the sum of current payments and current receipts (goods, services, income, and transfers) for the most recent five-year period and expressed as a share of the sum for all countries;
- **Variability** (v_i) is the variability of current receipts and net capital flows, measured as a standard deviation from the centered 3-year moving average over a recent 13-year period and expressed as a share of the sum for all countries;
- **Reserves** (r_i) is the 12-month average over the most recent year of official reserves—foreign exchange, SDR holdings, reserve position in the IMF and monetary gold—and expressed as a share of the sum for all countries; and
- **Compression** (k) is a compression factor equal to 0.95 applied to the uncompressed calculated quota shares, which are then rescaled to sum 100.

To get a better sense of how the formula works we can look at the contribution of each variable by economy type. Contributions are calculated as the ratio of a variable times its weight divided by the weighted sum of the variables prior to compression. Countries are classified following the *World Economic Outlook* criteria that divides the world into 39 advanced economies, 155 emerging market and developing countries (EMDCs) and 59 low-income developing countries (LICs)¹⁴

For the purpose of this analysis, Advanced economies are subdivided further into the seven largest according to GDP at market exchange rates (ADV_MAJ) and the remaining twenty-eight (ADV_OTHR). Data has a lag—the most recent available data for all countries at the time of research is 2019, released in 2021—and all variables are calculated as averages over longer periods of time to capture longer-term trends and to avoid sharp movements due to individual observations.

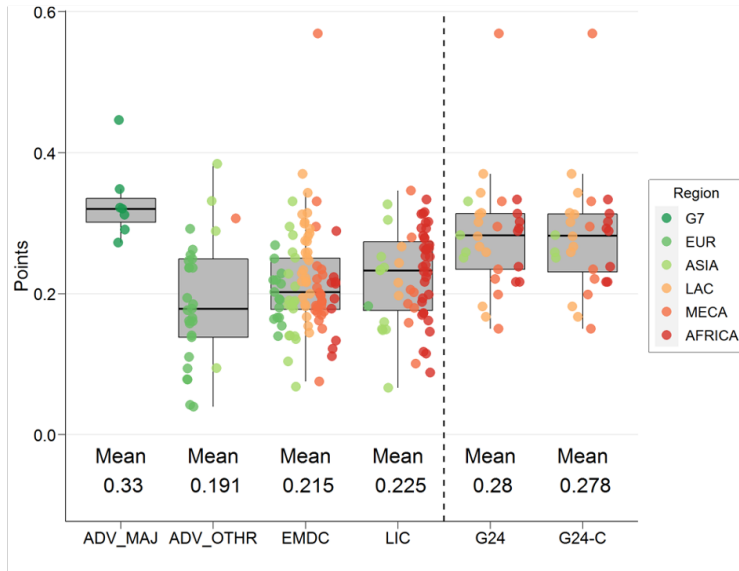
Figure 1 shows the contribution of the *GDP-blend* variables. It combines a box plot and a jittered plot of the contributions for each economy type with the value of the weighted average shown at the bottom¹⁵. For none of the four economy types the combined contribution of GDP is above 50 percent, but there are significant differences: a larger contribution from GDP at market rates for ADV_MAJ economies—generating over a third of their quota—while for EMDCs and LICs the largest contributor is GDP at PPP, with a contribution that is as large or larger than its weight in the formula. These differences reflect the fact that market prices of non-tradable goods tend to be lower in countries with smaller per capita incomes, something further confirmed by the low contribution of GDP at PPP to the quota of other advanced economies, and by the fact that the

¹³Variable definitions taken from [International Monetary Fund \(2018e\)](#).

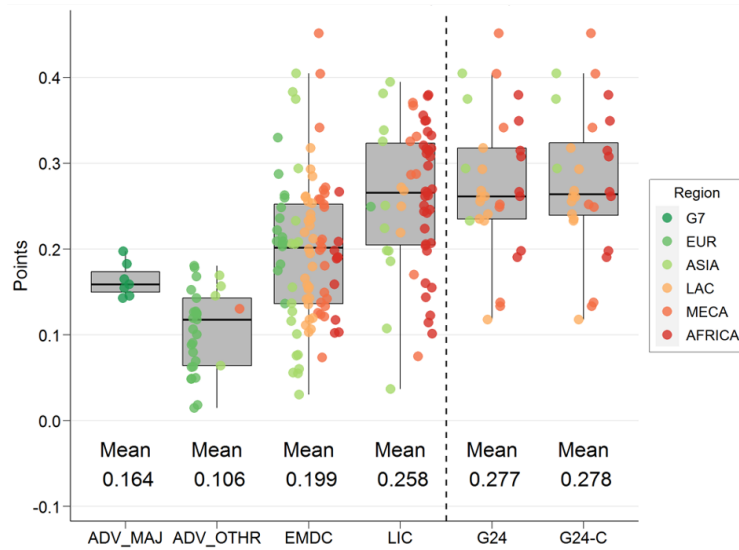
¹⁴Note the WEO classification has some differences with the one used in staff's quota analysis. See for example [International Monetary Fund \(2018f\)](#) Statistical Appendix for details.

¹⁵See Appendix C for a key to reading the box plots.

contribution of this variable is larger for LICs than for EMDCs. G-24 countries have a contribution of of the *GDP-blend* variables well above 50 percent as do G-24 members without China (G24-C).



(a) GDP at market prices



(b) GDP at ppp

Figure 1: Contribution of GDP-blend variables

The case of *openness* is different in the sense that it contributes more to the quota of smaller and advanced economies. Figure 2 shows it generates above 40 percent of the quota of the ADV_MAJ economies—a larger contribution than the GDP blend despite the latter's larger weight in the quota formula. This reflects the fact that many small economies have become very large hubs for trade and financial services relative to the size of their economy.

Luxembourg, for example, has an absolute ratio of Openness to GDP of 10.6, meaning that the sum of its current payments and receipts of goods, services, income, and transfers is almost 11 times larger than its GDP. Overall, as noted in [International Monetary Fund \(2018a\)](#), the top 15 countries in the Openness-to-GDP ranking have a ratio greater than 2, with eight of them being small advanced economies.

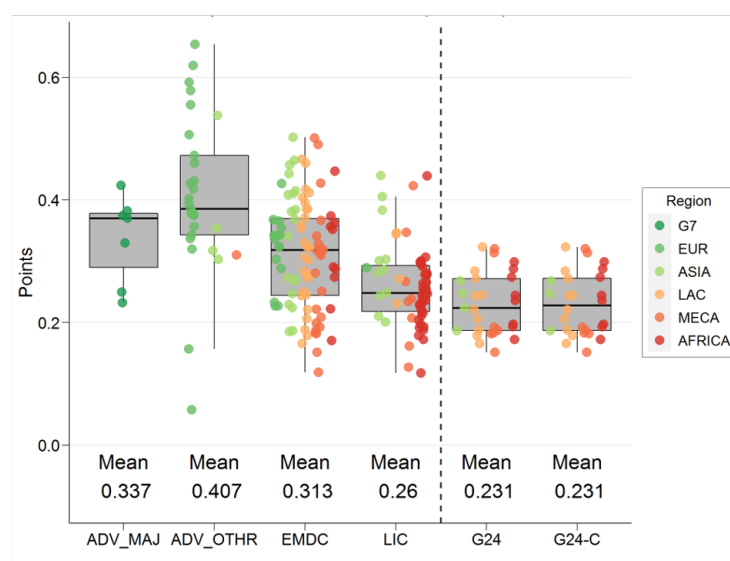


Figure 2: Contribution of openness

This has led to a variety of concerns regarding the measurement of openness given the disproportionate effect it has relative to the rest of their economic variables. Many of the criticisms point to the fact that it is measured in gross terms, thus leading to double counting of cross border flows when production is vertically integrated. Yet, since no simple solution exists given the limitations of the trade value-added data that could be used to overcome the issue, IMF staff has instead proposed placing a cap on openness or reducing its weight in the formula¹⁶.

Turning to *variability*, it is probably the most contentious variable in the formula. Although it is included to approximate the potential need for Fund resources by capturing a country's vulnerability to balance of payments shocks, work conducted by staff has failed to find such a relationship¹⁷. Furthermore, no better measurement has been identified, and a very high correlation with openness (0.98) and an almost identical distribution have been observed. Note that this implies smaller and more advanced economies benefit proportionally more from the inclusion of variability than other economy types as shown in figure (3).

Like openness, a large proportion of the top 15 countries in the Variability-to-GDP ranking are small advanced economies, with Iceland ranked top with a ratio of 69 percent. This has led many to call for its elimination from the quota formula, arguing that keeping it serves only to increase

¹⁶See for example [International Monetary Fund \(2018a\)](#).

¹⁷See for example [International Monetary Fund \(2018c\)](#).

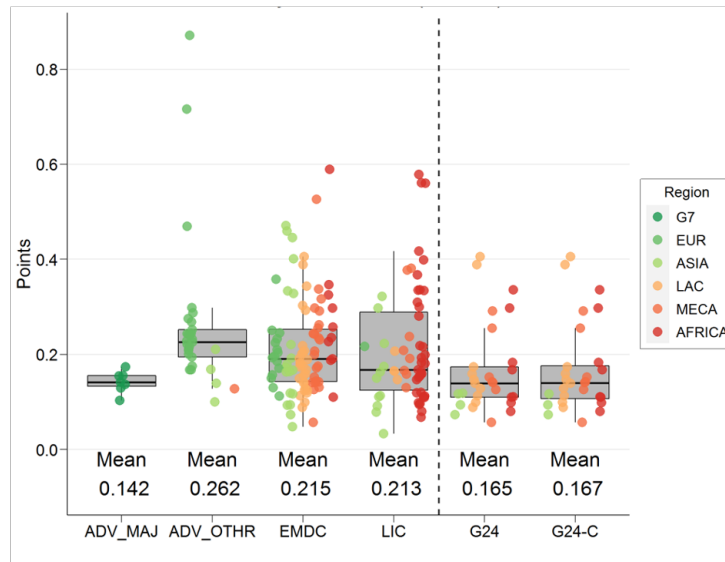


Figure 3: Contribution of variability

the weight of openness and exacerbate the problems associated with it.

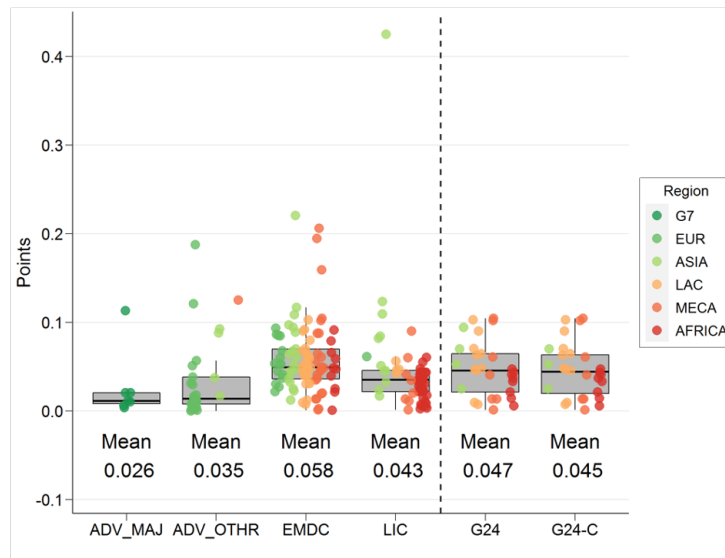


Figure 4: Contribution of reserves

Finally, *reserves* is the variable with the smallest weight. It was included to reflect members' potential to contribute to Fund's resources and therefore plays a different role in the quota formula. This is supported by the fact that its correlation to other variables is the lowest among all and that it benefits EMDCs the most as illustrated in figure 4.

Figure 5 summarizes this discussion, with average variable contributions for each economy

type, and the weight of variables in the current formula for comparison. The outsized role of openness for ADV_MAJ and the relative importance of GDP at PPP for EMDCs—particularly for G-24 countries—and LICs in supporting their quotas are apparent from the figure and a recurring topic in quota reform discussions. For G-24 countries the effect is even larger.

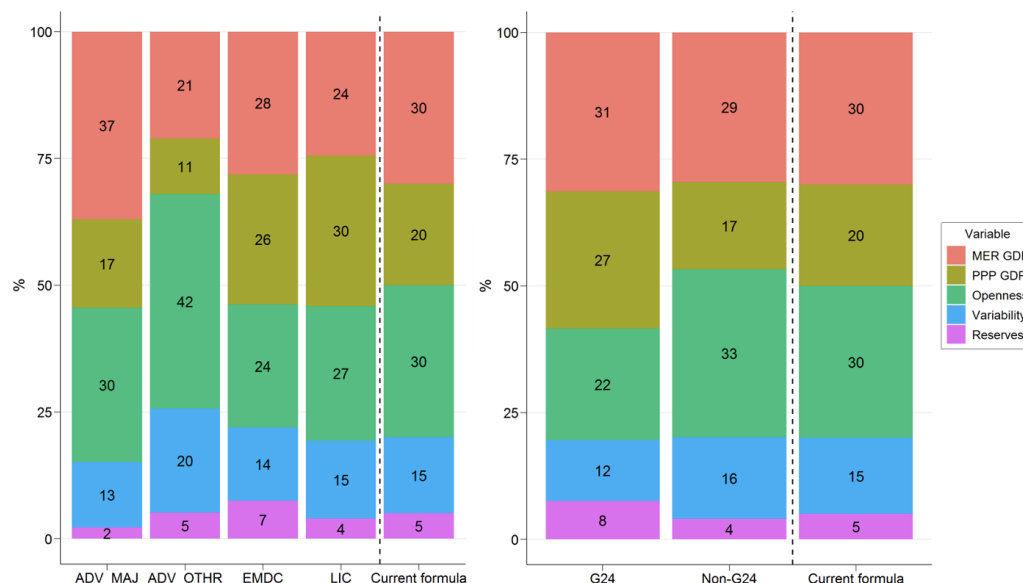


Figure 5: Summary of variable contributions

Once variables are weighted and added, the compression factor is applied to obtain final quota shares. Since $k < 1$ the effect of compression is to mitigate the effect of size in quotas by reducing it for the largest members and increasing it for the smallest. The resulting quotas are presented in figure 6. Note the information is plotted by Chair to reflect the way the Executive Board is organized to carry out discussions and vote¹⁸

The figure shows the calculated quota shares (CQS)—i.e. applying the latest data to equation 1—and, for comparison, the actual quota shares (AQS) that were agreed in the 14th review and that are used as ordering variable. Two facts stand out. First, the concentration in the distribution of quotas as evidenced by the fact that the 7 single country Chairs have 47 percent of current quotas, while the 46 countries grouped in the two African constituencies—a quarter of the members—have only 3.56 percent. This is further confirmed by the Gini coefficient of quotas across countries which has a value of 0.807¹⁹

¹⁸To facilitate the identification of the 24 Chairs, names for multi-country constituencies are given based on their largest two countries—e.g. SA_NIG for South Africa and Nigeria, or CONG_SEN for Democratic Republic of Congo and Senegal—except for the ones that take their name from their respective regions like EUR_TKY for Eastern Europe and Turkey, or SOUTH_AM for the South American constituency that includes Argentina, Peru and Chile. OEDCE, the Chair representing Colombia, Mexico, Spain, Venezuela and four Central American countries, is the only one taking its formal name. See Appendix D for details.

¹⁹The Gini for the distribution of quotas at the Chair level is 0.31, similar to the income distribution of Ireland, South Korea or Pakistan. This Gini reflects the fact that current rules for constituency formation are designed so Chair quotas fall within a predetermined range to limit size heterogeneity.

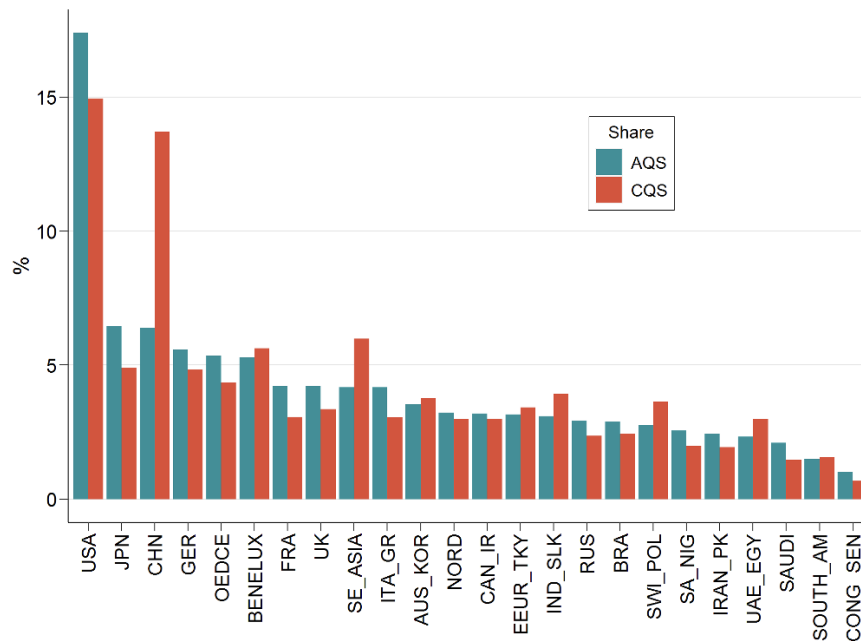


Figure 6: Actual quota shares and calculated quota shares

The second fact is the difference between actual and calculated quota shares. Although the differences do not seem to depend on country size or economy type, their pervasiveness points to a generalized misalignment. If the current formula does reflect appropriately the relative economic importance of a country and its ability to effectively contribute resources—an assumption strongly questioned by many—a significant adjustment would be necessary to bring actual quotas in line with calculated quotas. In any case quotas are unevenly split between developing and advanced economies, with EMDCs having 8 times the quota of LICs and ADV_MAJ being 2.4 times larger than other advanced economies (see figure 7).

III. CHANGING THE QUOTA FORMULA

Two main reasons are generally given in favor of changing the quota formula. First that, as mentioned above, the actual distribution of quotas is not an accurate reflection of the relative importance of members in the world economy; second, that the resulting distribution from applying the current formula fails to adequately address the problem. There is no agreement on how this could be addressed, but it is probably safe to say that most countries believe a change is required to address the issues.

Changing the formula implies changing its parameters to reach an outcome for which there is sufficient support. But given the disparity of interests among member countries, IMF staff has usually provided a large set of reform options that capture the diversity of the discussion, making it is easy to lose sight of the big picture and the underlying mechanisms driving specific results.

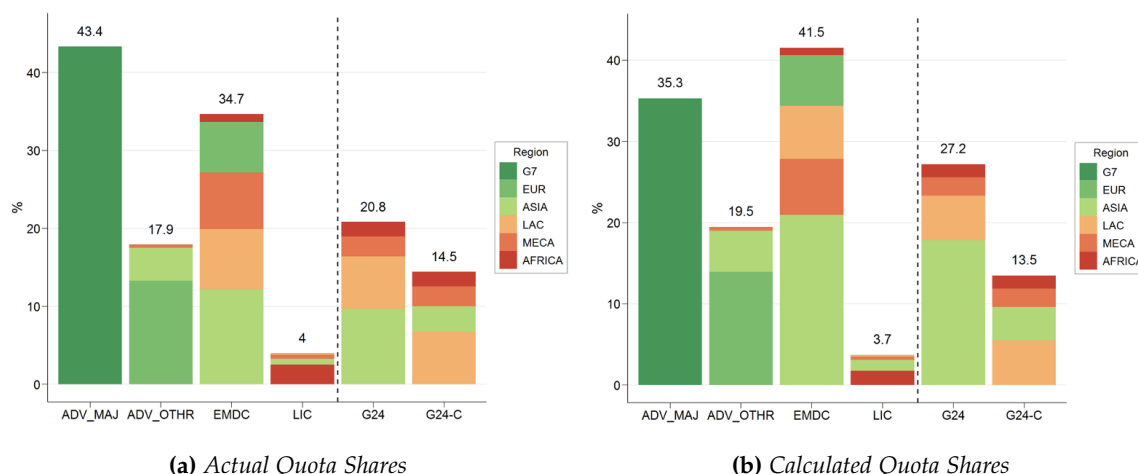


Figure 7: Distribution of quotas under the current formula

It is thus useful when looking at reform scenarios to identify which of the three possible sources of change are at play. They are (i) formula parameters, (ii) the weight of the specific formula being considered and (iii) the inclusion of non-formula components—i.e. voluntary financial contributions and protection of the poorest and smallest members.

We start with the first, which is the focus of most of the discussions and where the large divergence of positions can be most clearly seen. As noted in Appendix A, the quota formula introduced in equation (1) can be rewritten as

$$Q_i = (\alpha y_i + \beta o_i + \gamma v_i + \delta r_i)^k, \tag{2}$$

to illustrate four key properties of the quota formula that are critical to understand the impact of parameter changes:

- Property 1:** Increasing the weight of a variable will benefit a country if the contribution of that variable is greater than its current weight. This property implies that a country’s quota will increase in response to a parameter raise only if the contribution of the associated variable is larger than the value of the parameter. So for example, with the current data, this property says the United States will benefit from an increase in α since the contribution of GDP to its quota is $0.644 > 0.5 = \alpha$. Conversely, its quota would fall if the weight of variability increased given that the contribution of that variable is $0.103 < 0.15 = \gamma$.

Figure 8 plots the contribution of the formula variables for the 24 Chairs to illustrate this property. A dotted vertical line is drawn for each variable at the current weight to facilitate seeing which Chairs would benefit and which would lose from parameter changes.

Take again the case of giving more weight to the GDP blend: it would increase, for example, the quota of the Brazilian, Indian and South American Chairs, but reduce it for the Benelux, Nordic and Saudi Chairs. The last three would rather see an increase in the weight of openness, except for the Saudis who only benefit if more weight is given to reserves or variability²⁰. Figure 9 shows variable contributions for all economy types and G-24 countries.

²⁰Since this property is based on an approximation, caution must be exercised when making conclusions for Chairs

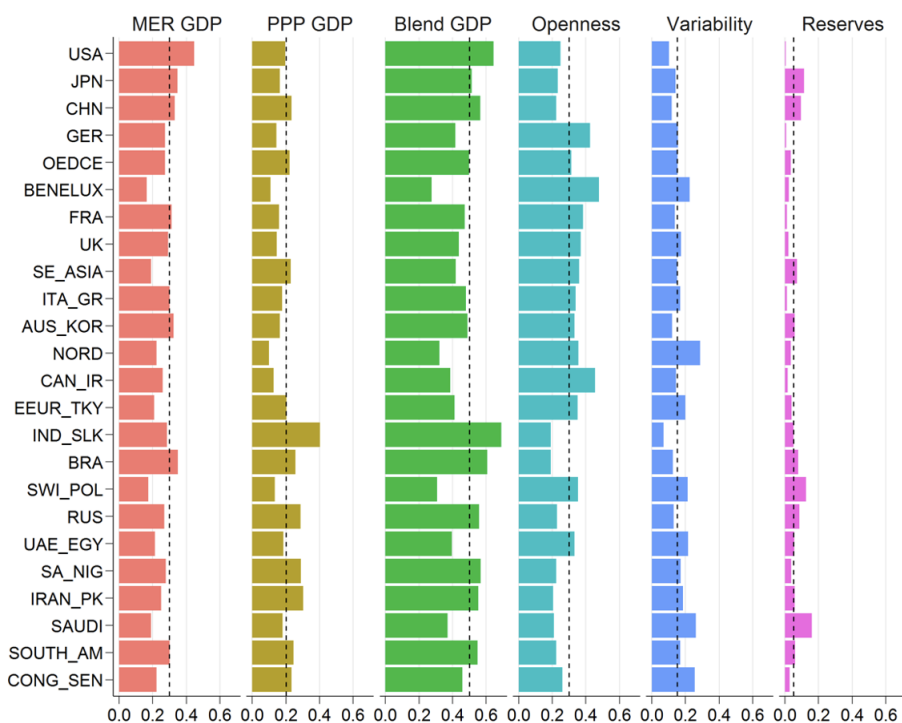


Figure 8: Variable contributions by Chair prior to compression

- Property 2:** *A country will prefer increasing the weight of the variable that has the highest level.* This property implies that among all of its variables, a country would prefer more weight be given to the one with the highest level. This is so even if it is not the variable that makes the largest contribution. As an example, take again Saudi Arabia: it would prefer more weight be given to reserves as it is the variable with the highest level (4.5) and not to GDP, despite the latter being the one that makes the largest contribution (0.37). The reason is that current weights "dilute" its strongest variable and "inflate" its weakest, but do not change the fact that for an equal increase in weight, reserves would have the biggest impact among all variables.

Although this property should be intuitively clear—give more weight to your strongest variable—it can be shown to follow from property 1. Figure 10 illustrates this property and shows that some Chairs like OEDCE or India-Sri Lanka have more homogeneous variable levels and thus more homogeneous preferences, while others like the Chinese or the Swiss-Polish have a variable whose level is disproportionately large when compared to the rest.

- Property 3:** *Increasing compression will reduce the quota of only the biggest members.* It can be shown there is a level of quota Q^* that determines the quota level for which reducing compression (increasing k) will have a positive effect on quota²¹. Below Q^* increases in k will reduce Q_i . This limiting value has no simple analytical form but can be calculated with

whose contribution is very close to the parameter value. That closeness, however, will mean the impact is very small and can end up having very limited practical significance. See Appendix A for details.

²¹See Appendix A for details.

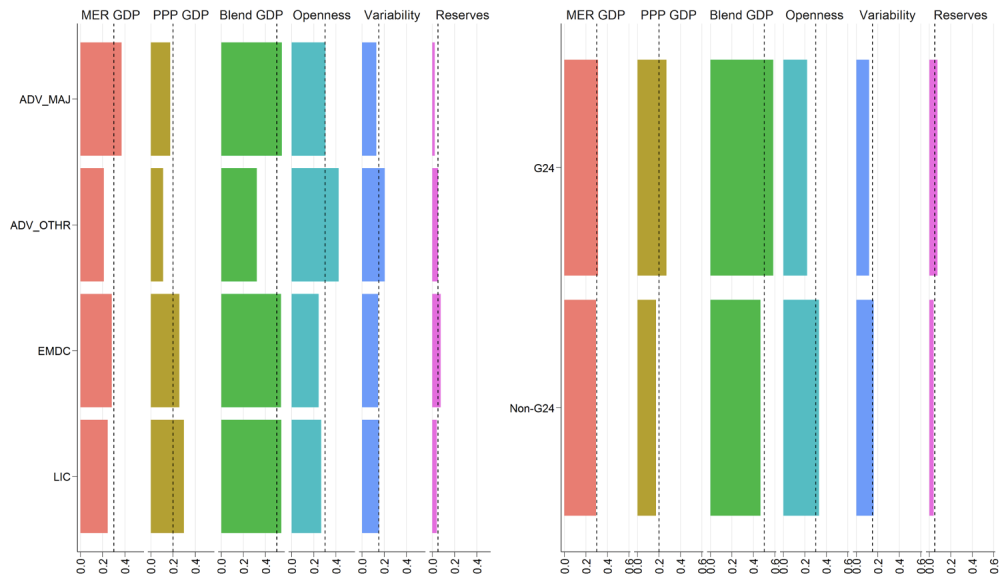
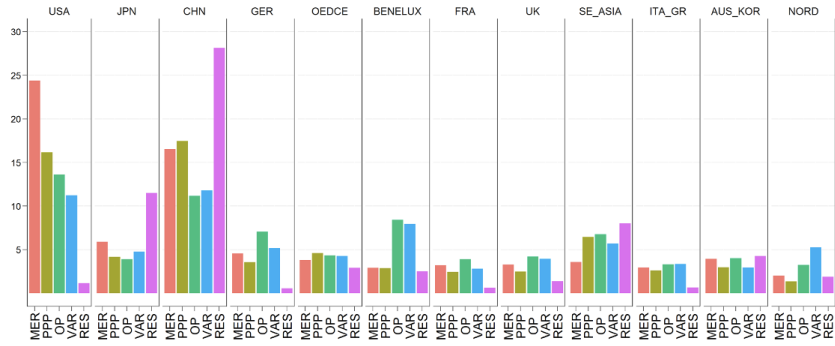
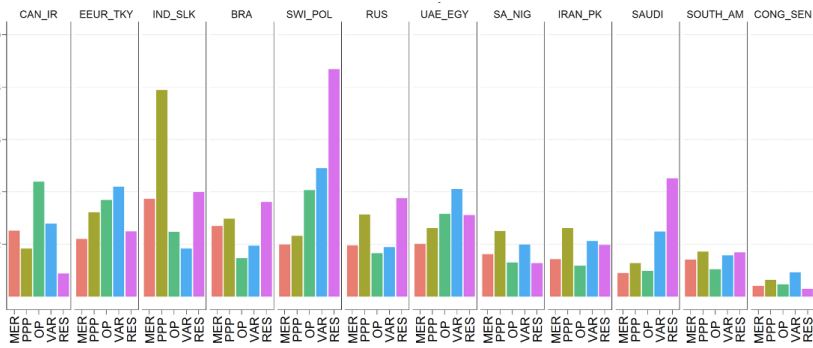


Figure 9: Variable contributions by economy type prior to compression



(a) top half by size



(b) bottom half by size

Figure 10: Variable levels

the current data to be roughly equal to $Q_i \gtrsim 2.4$. The implication is that only the nine largest countries (United States, China, Japan, Germany, France, United Kingdom, Italy, India and Russia) would see their quotas reduced by an increase in compression, while the rest will see theirs raised—including Brazil, the limiting country from below.

- **Property 4:** *Changing compression changes the spread of the distribution but not the ranking of countries.* Ranking invariance is an important property since the purpose of compression is to reduce concentration by transferring quota from the largest to the smallest members, but without creating the conflicts among benefited countries that could arise if their relative positions were altered.

Figure 11 illustrates properties 3 and 4 by plotting the percentage change in the calculated quota shares for positive and a negative changes in compression. As expected, the sign of the relative change between the largest nine countries and the rest differs, and its magnitude increases as we move to the extremes. Note the asymmetry in the impact of the change: a 5-basis points reduction k , for example, will increase the quota of the smallest member by almost 60 percent and reduce it for the largest by less than 10 percent—although absolute value reductions for the largest countries are obviously bigger.

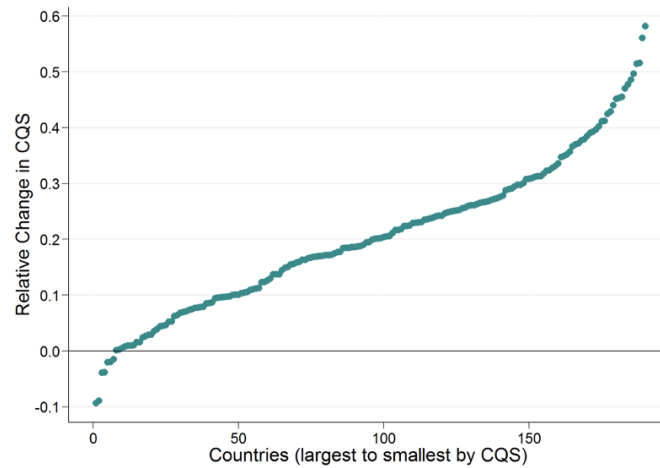
Beyond formula parameters, the second possible source for quota changes is the weight given to a specific formula being considered. Note there are two ways to allocate an quota increase: in proportion to the actual quota distribution (*equiproportional increase*) or according to the distribution that would result from a different formula (*selective increase*). The first will have no effect on the quota distribution as it uses the current distribution, but the second will alter it in a way that depends on how much the parameters of the new formula differ from the current ones, and on how large the increase is. So, for example, a doubling of quotas allocated using a new formula will result in a distribution that is a simple average of the current distribution and the distribution generated by the new formula; while a 50 percent increase would result in a distribution two thirds from the current formula and one third from the new formula.

The final source of change is the inclusion of non-formula components. The first are voluntary financial contributions (VFCs) to the IMF. They, which as the name indicates involve a provision of financial resources that is voluntary in nature, include Bilateral Borrowing Agreements, multi-lateral support for Fund liquidity through the New Arrangements to Borrow, loan contributions to poverty reduction facilities, subsidy contributions for concessional financing and capacity development. These are aggregated in different ways to produce three measures of VFCs whose allocation varies between 68–79 percent to advanced economies and 21–32 percent to EMDCs²². VFCs are usually included in a quota increase by assigning first a fraction of the total quotas for this purpose, and then distributing it in proportion to the country shares in the contributions.

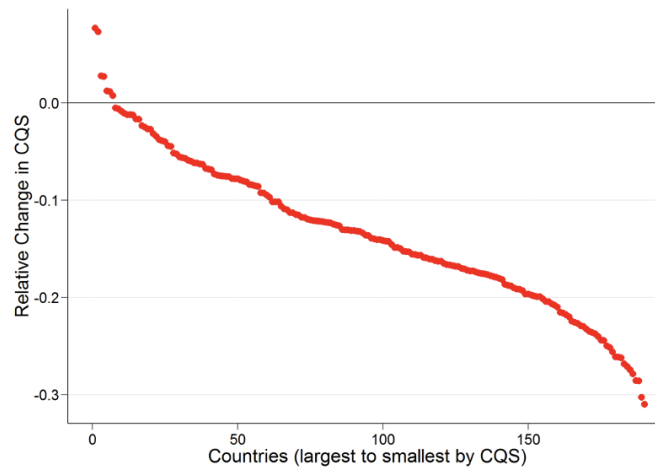
The other non-formula component is the protection of the poorest and smallest members. There are several possible definitions of poorest members, but in the 14th review they were defined as "PRGT-eligible countries with annual GNI per capita below the prevailing operational IDA cut-off point in 2008 (USD \$1,135) or below twice the IDA's cut-off for countries meeting the definition of a 'small country' under the PRGT eligibility criteria"²³. Other possible definitions that have been

²²See [International Monetary Fund \(2018c\)](#) and [International Monetary Fund \(2018a\)](#) for details.

²³[International Monetary Fund \(2018a\)](#) page 27. PRGT is the Poverty Reduction and Growth Trust that allows low-income



(a) reducing k to 0.90.



(b) increasing k to 0.99.

Figure 11: Distributional effects of changes in compression

considered are the list of all currently PRGT-eligible countries, the United Nations list of Least Developed Countries, and the WEO's list of Low Income Developing Countries²⁴²⁵.

In the case of small states, they are defined as developing countries with population below 1.5 million²⁶. Taken together the group of the poorest and smallest members has 84 countries, but numbers could vary if a different definition than PRGT eligibility is used²⁷. The cost of protection

countries to borrow in concessional terms. IDA or International Development Association is a part of the World Bank that aims to reduce poverty by providing special access loans and grants to the most vulnerable countries.

²⁴See [International Monetary Fund \(2018a\)](#) for details.

²⁵Note that the actual level of quotas required to prevent a fall in the quotas of the poorest members is relatively small. A change from actual quotas to calculated quotas, for example, would reduce the quotas of LICs by 0.3 percentage points as shown in figure 7.

²⁶See [International Monetary Fund \(2017\)](#) for details.

²⁷See [International Monetary Fund \(2018a\)](#).

is calculated as the share of the quota increase required to maintain the quotas of these countries; and, depending on the reform scenario being considered, it is relatively low with a range between 0.8 and 1.6 according to staff's estimates²⁸.

IV. IS AN AGREEMENT LIKELY?

Up to this point the emphasis has been on the current formula and in understanding its response to parameter changes. We now turn to the reform options being considered as part of the 16th review, and explore whether an agreement is likely under the current circumstances.

The first and fundamental problem for reaching an agreement are the distributional consequences of reform. This is the case because any change is akin to redistributing a pie of size 100—a zero-sum game in which any increase in the quota of a member has to be at the expense of the quotas of other members. Getting over the 85 percent level of support needed to change the formula implies finding a way for losers to agree, either through commitment to a set of overarching principles and/or via coercion or compensation mechanisms.

The first step in approaching this problem is to consider the difference between actual and calculated quota shares or *out-of-lineness* (OL) which is usually expressed as the ratio of CQS to AQS—with values above 1 implying a country is underrepresented—or as the absolute difference between the two. For both cases the calculation is made under the assumption that the current formula correctly expresses both the position of a country in the world economy, a claim strongly challenged by many of the out-of-line members. In any case, this definition of OL is consistent with the consensus reached in agreeing to the current formula in 2008.

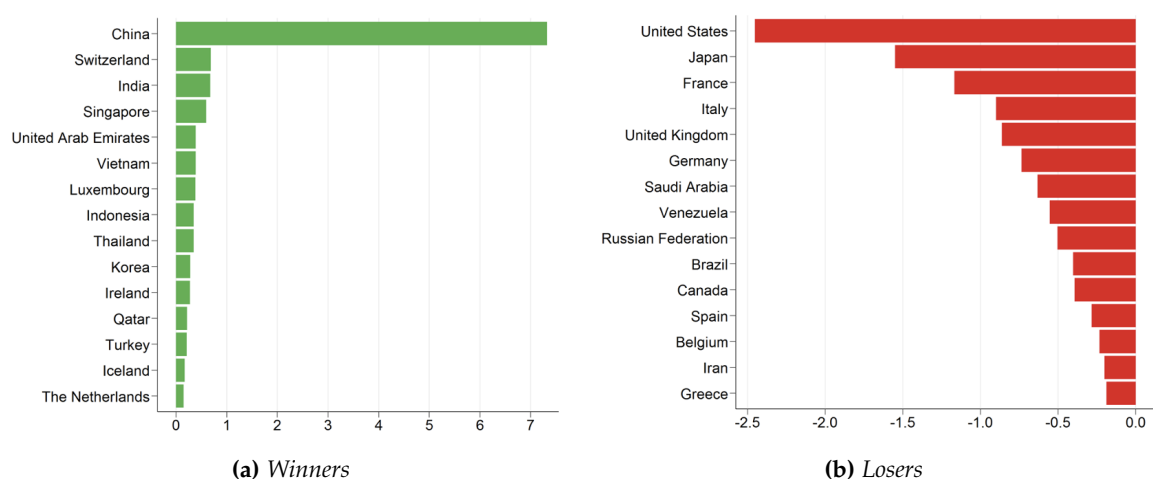


Figure 12: *Out-of-lineness*

Looking at the data by country gives a more granular sense of the magnitude of the problem and highlights the difficulties of closing the gap. Figure 12 shows who the main winners and losers would be if quotas were to move from the actual distribution to the one implied by the current formula. Overall total OL is 12.44 percentage points or roughly one eighth of quotas,

²⁸See [International Monetary Fund \(2018d\)](#).

indicating the magnitude of the transfers required to address the issue. The left panel shows the largest recipients, with China alone accounting for almost half of the transfers and the top fifteen countries for 87.5 percent. On the other hand, the countries that would need to give up quota are led by the United States with a reduction of 2.47 percentage points, while the top fifteen account for 78.2 percent of the required reductions.

This is where most of the political economy problems arise. A large proportion of the misalignment could be solved with a transfer of quotas from the countries in the right panel to the countries in the left panel, provided sufficiently compelling reasons are found to get the support for such transfers. But even in that case an important complication arises from the fact that a reduction of 2.47 percentage points would leave the US without the veto power it has to block this type of changes, and by the fact that more than half of the top winners are high-income economies—something that goes against the spirit of increasing the voice and representation of the poorest members. So what does this imply for the likelihood of reaching an agreement?

The short answer is that a full shift to calculated quotas would probably be impossible. A more gradual change is likely the only way forward. The Executive Board has recognized this since the 15th review and instructed staff to carry out a wide set of simulations that can capture members' main concerns and provide a basis for reform. They follow a two-step process in which a series of formulas involving different parameter changes are defined first, and then possible allocations simulated.

In the first step permissible ranges of variation for parameter values are established to limit the number of scenarios. They include a basic set in which variability is dropped and four options considered for splitting its weight between GDP and openness, whose weights become (0.575/0.375, 0.6/0.35, 0.65/0.3 and 0.7/0.25). This set is then combined with three additional sets: one with three new weights for GDP at market prices and PPP (50/55, 55/45 and 65/35); another with four different caps on openness (85th percentile, 75th percentile, share equal to 1.8 and share equal to 1.5); and one with two compression values (0.925 and 0.975).

This produces 36 scenarios which are then complemented with an additional three that allow a greater role of GDP²⁹. The latter three include a GDP-only formula with current weights that reflects the view of those who think GDP is the only variable needed to capture relevant economic importance; and a midpoint formula between GDP-only and the current formula that aims to reconcile the views of those who favor giving all the weight to GDP and those who think the current formula does a better job at capturing a country's economic relevance. They materialize in sets A, B and C each allowing for an increasing range in parameter variation³⁰. Figure 13 summarizes all the scenarios considered.

As expected, the impact on a country's quota of a reform scenario will depend on the specific parameter changes considered and in the variable contributions for the country—i.e. the formula properties discussed in the previous section. Figure 14 shows the percentage change in quota for all reform scenarios. As a group, EMDCs will benefit in all scenarios, ADV_MAJ economies will always lose and LICs and ADV_OTHR will have a mixed picture (left panel). Note that for the later

²⁹See Annex II in [International Monetary Fund \(2018a\)](#).

³⁰Among the possibilities examined for each set, the one chosen minimizes the absolute distance to the midpoint. That is, the three sets are used to identify the formula parameters that produce the closest distribution to the midpoint under the belief that they could provide an acceptable compromise between current-formula and GDP-only views.

SET 1: Simplification of the current formula by dropping variability, keeping current GDP and openness measures:												
Alternative formulas: distribution of variability weight to GDP and Openness												
57.5 BI - 37.5 Op			60 BI - 35 Op			65 BI - 30 Op			70 BI - 25 Op			
SET 2: Same as Set 1, but with different compositions of GDP blend:												
Alternative formulas: distribution of variability weight to GDP and Openness, and different GDP blends (MER/PPP)												
57.5 BI - 37.5 Op			60 BI - 35 Op			65 BI - 30 Op			70 BI - 25 Op			
50/50	55/45	65/35	50/50	55/45	65/35	50/50	55/45	65/35	50/50	55/45	65/35	
SET 3: Same as Set 1, but with different openness measures:												
Alternative formulas: distribution of variability weight to GDP and Openness, and different caps on Openness												
57.5 BI - 37.5 Op			60 BI - 35 Op			65 BI - 30 Op			70 BI - 25 Op			
85 th percentile	75 th percentile	Shares cap 1.8	shares cap 1.5	85 th percentile	75 th percentile	Shares cap 1.8	shares cap 1.5	85 th percentile	75 th percentile	Shares cap 1.8	shares cap 1.5	
SET 4: Same as Set 1, but with different degrees of compression:												
Alternative formulas: distribution of variability weight to GDP and Openness, with different degrees of compression												
Higher Degree of Compression (0.925)						Lower Degree of Compression (0.975)						
57.5 BI - 37.5 Op			60 BI - 35 Op			65 BI - 30 Op			70 BI - 25 Op			
57.5 BI - 37.5 Op			60 BI - 35 Op			65 BI - 30 Op			70 BI - 25 Op			
SET 5: Middle-Point approach:												
SET A: Restrictions Based on Previous Formula Simulations $(0.70 \cdot \text{GDP} + 0.25 \cdot \text{Openness} + 0.05 \cdot \text{Reserves})^{0.975}$, 60/40 GDP Blend												
SET B: Wider Range for GDP, Openness, GDP Blend and Compression $(0.75 \cdot \text{GDP} + 0.20 \cdot \text{Openness} + 0.05 \cdot \text{Reserves})^{0.975}$, 65/35 GDP Blend												
SET C: Allowing Reserves to vary $(0.75 \cdot \text{GDP} + 0.225 \cdot \text{Openness} + 0.025 \cdot \text{Reserves})^{0.975}$, 60/40 GDP Blend												

Figure 13: Simulation sets used to examine formula changes

the best outcome would be CQS and will thus give them an incentive to defend the status quo (CQS).

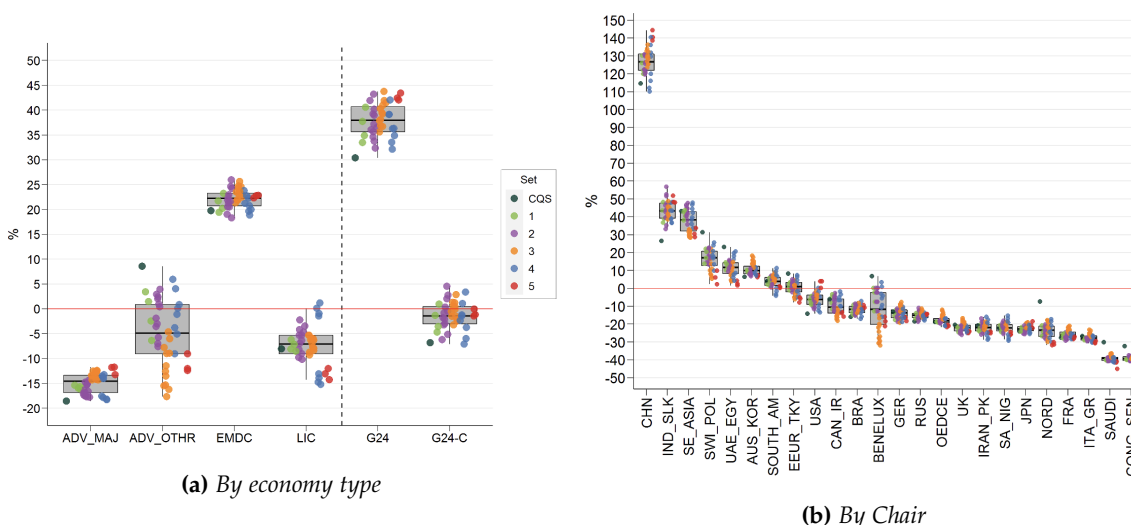


Figure 14: Changes in Quota for for all reform scenarios

Along the same lines, of the 24 Chairs six will always increase their quota shares, 14 will

always see a reduction and for the remaining 4 the impact will depend on the specific scenario considered (Right panel). A similar picture emerges for individual countries as can be seen in figure 15. Consensus, as predicted, is very difficult in a zero sum game.

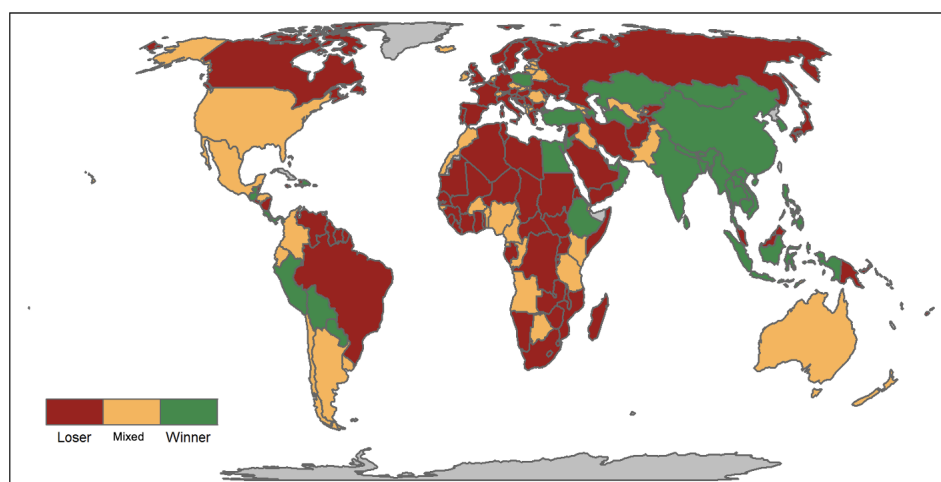


Figure 15: Winners and losers in all Sets

Despite this difficulty, it is important to determine individual preferences as they can facilitate the negotiation process. Figure 16 shows the ranking for the 39 allocation scenarios plus the actual (14th Review) and calculated (CQS) quota shares. Preferences are based on the difference between current quotas and the resulting quotas for each of the allocation scenarios, and ranked row-wise for each Chair from largest to smallest, with a darker color indicating greater preference³¹.

The key message from this figure is that there are no obvious universal points of convergence. Dark tones do not cluster around specific scenarios, except for the current quota distribution which is the preferred option for almost half of the Chairs—including seven of the ten largest—a strong deterrent for reform. Yet there seems to be some convergence for the smaller chairs around some Set 4 scenarios, and for the bigger chairs around some of Set 3 scenarios. This should come as no surprise since the smallest members benefit from increases in compression, while the biggest tend to benefit both from dropping variability and capping openness.

We can go a step further by exploring how preferences translate into support for specific scenarios. To do this we can simulate voting by first assuming Chairs would give a favorable vote to those scenarios that improve their quota, and a negative one to those who reduce it. Evidently there can be discrepancies among countries within a specific Chair, so we assume a majority view prevails akin to having a vote inside the constituency so that a scenario is supported if it benefits a majority of the Chair.

We also consider the implications if chairs were willing to accept reductions in their quotas. Figure 17 shows the results of voting simulations under different assumptions, with total votes equal to the sum of the actual quota shares of the countries that voted in favor. Four possibilities

³¹For every Chair allocation scenarios are given an index number from 1 to 41 and ties assigned their average index number. Chairs on the vertical axis are ordered from top to bottom by the size of their current quota.

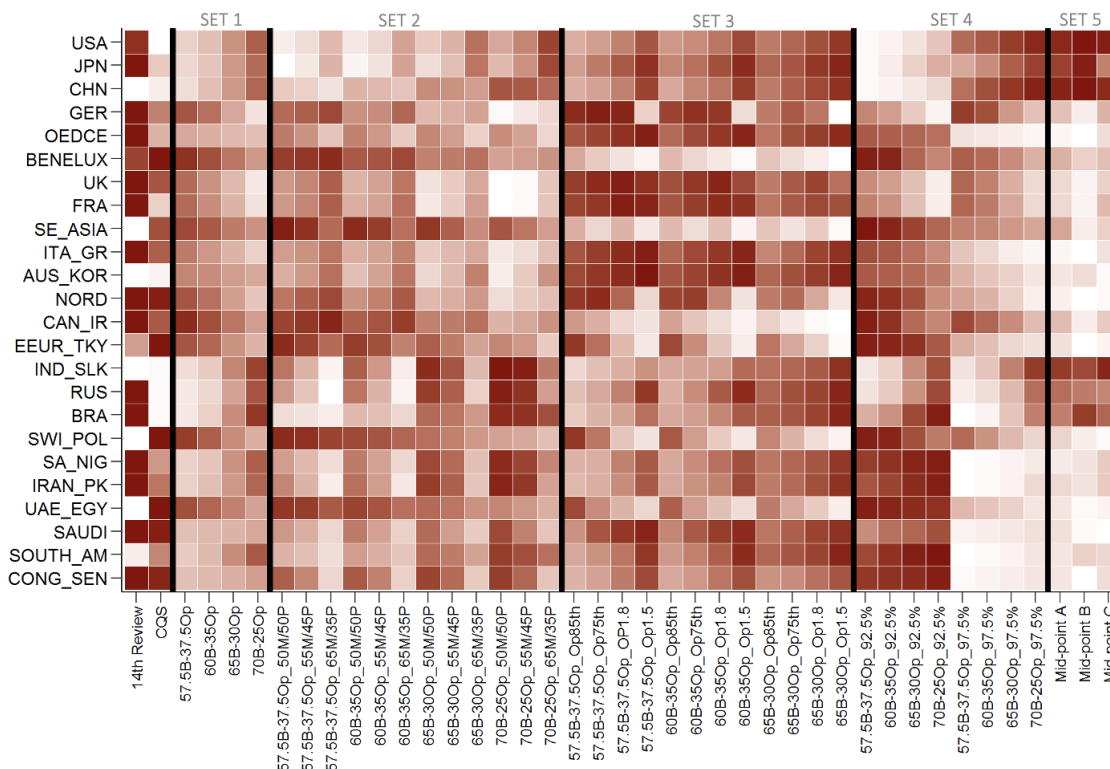


Figure 16: Reform scenario ranking by Chair (darker=greater preference)

are considered, ranging from Chairs voting only for scenarios that don’t reduce their quota (DQS > 0%) to Chairs accepting—i.e. voting for—scenarios that cause a maximum reduction of 10 percent in their quota share (DQS > -10%)³².

Support for all scenarios is pretty homogeneous when no reductions in quota are accepted. It revolves around 25 percent—except for the three *Mid-Point* scenarios and the Set 4 scenario that reduces compression and increases the weight of GDP to 70 percent that get slightly above 42 percent—far from the required 85 percent mark represented by the vertical dotted line. Allowing for a 1 or a 5 percent reduction does not cause any significant changes. In the end, not even with a 10 percent reduction it is possible to get to 85 percent support for any scenario. This reiterates how difficult it will be to reach an agreement based on the formula alone, and underscores the necessity of finding ways to convince a significant part of the membership to accept quota reductions.

While potential support is the most important aspect in judging possible reform scenarios, it is not the only one. At least two more are explicitly stated as goals of the quota review: adequately reflecting relative positions in the world economy and ensuring voice and representation of the most vulnerable members. Improving the quota share of these countries and reducing out-of-lineness are therefore concrete ways to measure to what extent reform scenarios advance

³²Note that the reduction values used for the simulations are arbitrary since there is no indication countries would ex-ante be willing to accept any of them. Also note that 10 percent was chosen as the the maximum permissible fall since it would still preserve United States’ veto power.

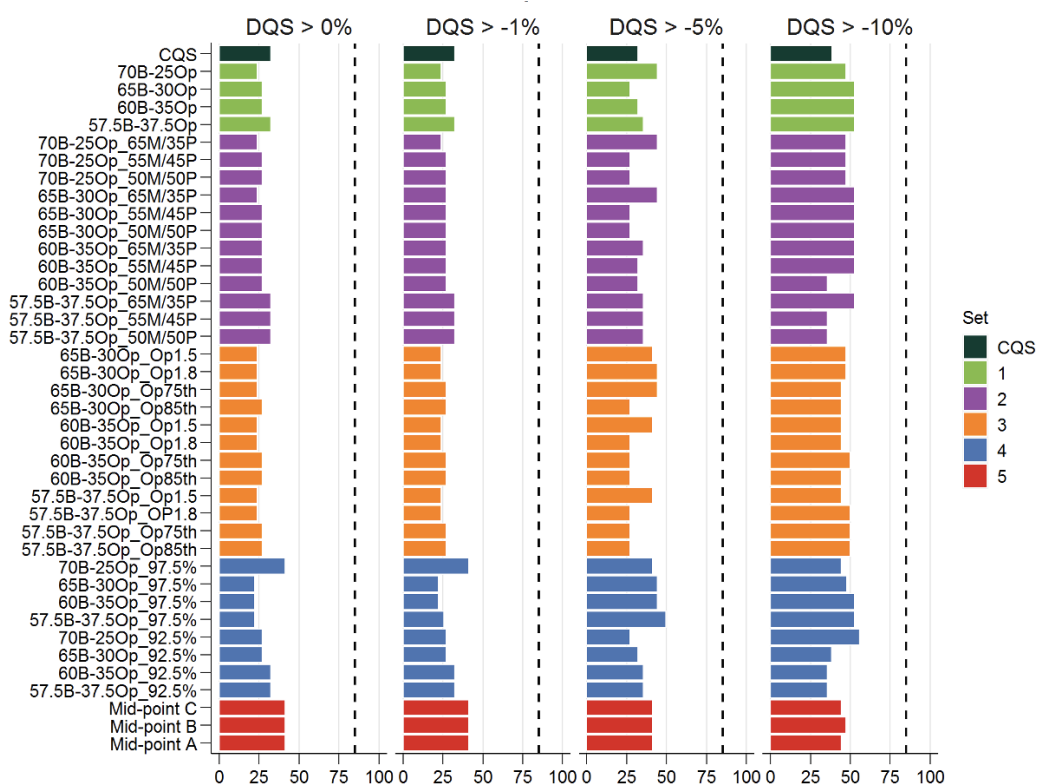


Figure 17: Simulated votes by reform scenario

these goals. Figure 18 explores the issue and examines whether trade-offs exist between support, misalignment and concentration. The horizontal axis measures the reduction in out-of-lininess that can go from no correction under the current distribution, to full correction under the current formula (CQS). The vertical axis measures the Gini coefficient at the country level for each scenario, with the red dotted line indicating the current Gini. Bubble sizes indicate scenario support without allowing for any reduction in quota.

The negative slope of the regression line suggests there is no trade-off between equity and misalignment. This is the case because larger reductions in out-of-lininess imply proportionally larger transfers of quotas to smaller members. Support, however, goes in the opposite direction with the largest number of votes going to the Mid-point scenarios that favor the largest countries—i.e. the ones that currently hold the largest voting power. Note, however, that none of the 39 scenarios reduce the current Gini indicating that a component for the protection of the poorest must be included via out-of-formula components to ensure distributional objectives are met while, at the same time, potentially increasing member’s support³³.

Given the difficulties of reaching an agreement and the complexities of the underlying process, it is important to consider if groups of Chairs can be formed that could potentially facilitate the

³³Other scenarios can be proposed that better align the interest of specific country groups. For example, a reform scenario consisting of GDP Blend 75% with equal weights to MER GDP and PPP GDP; openness 20% with a cap of 1.5; no variability; reserves 5% and a compression factor of 0.925 would get almost unanimous support from G-24 members.



Figure 18: Distribution, out-of-lininess and reform support

negotiation process. As a final point we do a preliminary exploration of this issue by conducting a simple cluster analysis based on variable contributions. Contributions are selected as the clustering variable since they are ultimately responsible for Chair positions towards reform proposals (See section III). Figure 19 shows the resulting dendrogram plot 34.

By looking at the figure it should be apparent that several groupings can be made. The figure is drawn for 6 clusters but, as is common with this type of analyses, statistical tests were inconclusive in indicating their optimal number. In any case it suggests at least three seem to exist, with one grouping the Chairs that benefit from reforms that give greater weight to openness and less to GDP—mostly European and open middle-income economies—another grouping the Chairs that benefit from more weight to GDP and less to openness—including the three largest members and some of the biggest economies—and a single-leaf cluster formed by Saudi Arabia whose dissimilarity stems from the fact that it is a Chair whose quota falls when more weight is given either to openness or GDP.

The first cluster can probably be subdivided further in two according to how much its members benefit from variability—with the Nordic, Benelux and Swiss Chairs being strong beneficiaries. The second could also be subdivided in terms of how strong the preference that more weight be given to one of the GDP blend variables, with the Russian, Indian, Iranian and South African Chairs strongly preferring more weight be given to GDP at PPP; and the rest not having very strong preferences—with the exception of the US that strongly favors GDP at market prices. There thus seems to be scope for clustering negotiations around variable-weight defined interest groups that could provide a basis to increase process speed and facilitate consensus.

³⁴Clustering was carried out using the complete linkage algorithm to obtain more compact clusters. Similarity is represented by vertical distance: shorter distance = more similar.

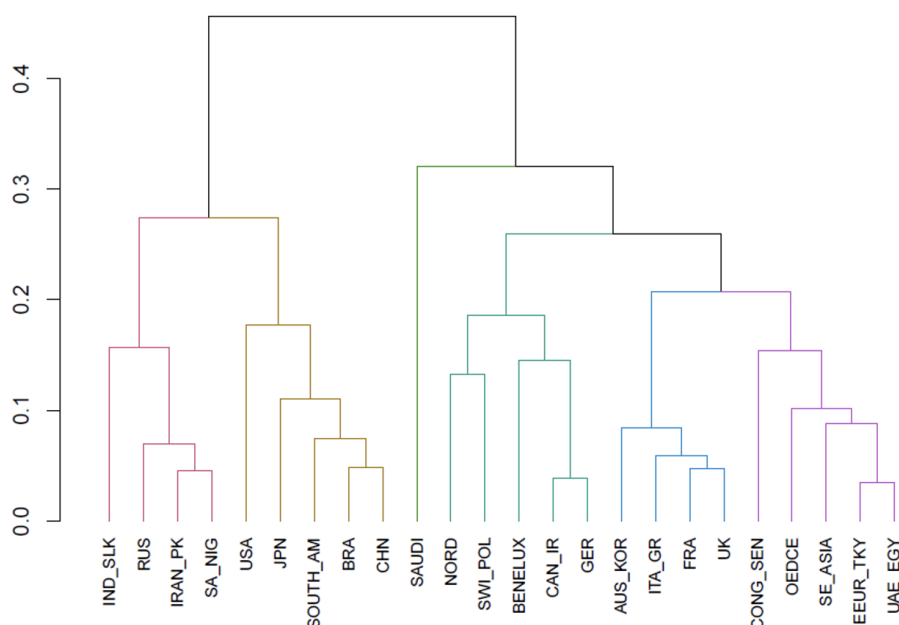


Figure 19: Dendrogram for variable contributions ($k = 6$)

V. THE NEXT AGREEMENT

Given all of the above, will there be an agreement? Trying to answer this question conclusively would require a game theoretic approach that is beyond the scope of this paper.³⁵ However, we can point again to the fact that for eleven chairs, including seven of the ten largest, the status quo is the preferred option. This is a strong force against agreement even if they only represent around 40 percent of the votes.

But we must also consider forces that favor reform like the current threat of recession that is causing turmoil in emerging markets, or the risks highlighted in the WEO that may convince many of the Chairs favoring the status-quo that demand for Fund resources may be higher than anticipated, and that the IMF is probably the least costly option to deal with future crises. Plus the fact that the only member with veto power should not be in principle opposed to reform as it prefers at least three of the proposed scenarios over the current distribution.

So any agreement will probably have to include a wider set of issues to pass the 85 percent mark. Approval of the 14th review, for example, included a doubling of quotas, a tripling of basic votes, limits to dilution in order to protect the most vulnerable members, a shift in quotas of 6 percent to acknowledge the growing importance of EMDCs, allowing bigger constituencies to appoint a second Alternate Executive Director, moving to an all elected Board and agreement to reduce the representation of advanced European economies by two Executive Director positions.

In the end, the inescapable reality is that reforming the formula is a zero-sum game where

³⁵Several attempts to explain the strategic interaction among Fund members have been made in the literature. See for example [Lipsy \(2015\)](#) or [Blomberg and Broz \(2013\)](#).

outside elements will have to come in to play for consensus to be reached. Quota reviews will continue being a political process anchored in a formula that serves to frame the discussion and provide legitimacy. We will probably arrive at a combination of increased resources, redistribution of quotas and a partial advance in Fund's goals facilitated by a set of external factors.

Such agreement will almost certainly be expressed as the sum of the quota formula plus an ad hoc component. Hopefully countries will be able to look beyond small reductions in their quota and focus on strengthening global financial stability by supporting an agreement that strengthens the IMF, ensures its legitimacy and relevancy, and provides the necessary tools to increase its effectiveness.

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APPENDICES

A. FORMULA PROPERTIES

The impact of parameter changes is not straightforward because introducing compression requires normalization to ensure quota shares add up to 100. This makes every member's quota depend not only on its formula variables, but also on the formula variables of the other 188 members. Equation (1) in the main text can thus be modified to include normalization and be rewritten as

$$Q_i = \frac{V_i}{S}, \quad (\text{A.1})$$

where $V_i = (\alpha y_i + \beta o_i + \gamma v_i + \delta r_i)^k$ and $S = \sum_{i=1}^n V_i$. Writing it like this helps illustrate four key properties of the quota formula that are critical to understand the impact of parameter changes:

- **Property 1:** *Increasing the weight of a variable will benefit a country if the contribution of that variable is greater than its current weight.* To see why we first calculate the derivative of (A.1) with respect to GDP weight

$$Q'_{\alpha i} = \frac{V'_{\alpha i} S - S'_{\alpha i} V_i}{S^2}, \quad \text{with } V'_{\alpha i} = k y_i (\cdot)_i^{k-1} \quad \text{and} \quad S'_{\alpha i} = \sum_{i=1}^n V'_{\alpha i}.$$

If we let $(\cdot)_i = (\alpha y_i + \beta o_i + \gamma v_i + \delta r_i)$ be the linear combination of the variables (LC) prior to compression, we can use this expression to determine when a parameter change will have a positive impact on the quota

$$\begin{aligned} Q'_{\alpha i} > 0 &\Rightarrow V'_{\alpha i} S > S'_{\alpha i} V_i \\ &\frac{V'_{\alpha i}}{V_i} > \frac{S'_{\alpha i}}{S} \\ &\frac{k y_i}{(\cdot)_i} > \frac{S'_{\alpha i}}{S} \\ &\frac{\alpha y_i}{(\cdot)_i} > \alpha \left[\frac{S'_{\alpha i}}{S k} \right]. \end{aligned} \quad (\text{A.2})$$

The right hand side of (A.2) does not provide a simple analytical solution as both S and $S'_{\alpha i}$ have 189 terms. But by shifting to a numerical approach we can simplify the term in square brackets by using the fact that $[S'_{\alpha i}/S k] \approx 1$. The inequality thus becomes

$$Q'_{\alpha i} > 0 \quad \Rightarrow \quad \frac{\alpha y_i}{(\cdot)_i} \gtrapprox \alpha. \quad (\text{A.3})$$

Note that an expression like (A.3) can be derived for all the formula variables since they enter the quota formula in a similar manner and the derivatives have the same functional form. With the current data we can verify that the values for the term in square brackets are 0.9901 for *GDP*, 1.0075 for *openness*, 1.0187 for *variability* and 1.0002 for *reserves*. Equation (A.3) will therefore hold as its right hand side becomes $0.495 \approx 0.5$ for *GDP*, $0.302 \approx 0.3$ for *openness*, $0.153 \approx 0.15$ for *variability* and $0.05001 \approx 0.05$ for *reserves*.

- **Property 2:** *A country will prefer increasing the weight of the variable that has the highest level.* Although this property should be intuitively clear—give more weight to your strongest variable—it can be shown to follow from property 1. By choosing two variables, say GDP and openness, we can determine increasing the weight of which has the largest positive impact on the quota of a given member:

$$\begin{aligned} Q'_{\alpha i} > Q'_{\beta i} &\implies V'_{\alpha i} S - S'_{\alpha i} V_i > V'_{\beta i} S - S'_{\beta i} V_i \\ &(V'_{\alpha i} - V'_{\beta i}) S > (S'_{\alpha i} - S'_{\beta i}) V_i \\ &\frac{y_i}{(\cdot)_i} - \frac{o_i}{(\cdot)_i} > \left[\frac{S'_{\alpha i}}{S k} - \frac{S'_{\beta i}}{S k} \right]. \end{aligned}$$

For the last line we rely on the fact that $V'_{\alpha i}/V_i = {}^k y_i / (\cdot)_i$ and $V'_{\beta i}/V_i = {}^k o_i / (\cdot)_i$ used in the derivation of property 1. We can simplify further by recalling each of the two terms inside the square bracket is approximately equal to 1 as established in the derivation of (A.3). Therefore

$$Q'_{\alpha i} > Q'_{\beta i} \implies y_i > o_i. \quad (\text{A.4})$$

By transitivity the analysis can be repeated to show that the variable with the largest derivative will be the one that has the highest level, and thus the one that will have the biggest impact on quota for a given weight increase.

- **Property 3:** *Increasing compression will reduce the quota of only the biggest members.* Intuitively equation (1) would suggest that the cutoff point in sign for a change in compression should be a quota level of 1 as its derivative with respect to k is a logarithmic function. However, this is not the case. To see why, note that

$$Q'_{ki} = \frac{V'_{ki} S - S'_{ki} V_i}{S^2}, \quad \text{with } V'_{ki} = V_i \ln(\cdot)_i \quad \text{and} \quad S'_{ki} = \sum_{i=1}^n V_i \ln(\cdot)_i. \quad (\text{A.5})$$

Dropping the indices in the summation, and writing $\ln(\cdot)_i$ as \ln_i , yields:

$$\begin{aligned} Q'_{ki} > 0 &\implies (V_i \ln_i S - \sum V_i \ln_i V_i) \frac{1}{S^2} > 0 \\ &\ln_i S > \sum V_i \ln_i \\ &(\cdot)_i > \exp(\sum V_i \ln_i)^{1/s} \\ Q_i &= \frac{(\cdot)_i^k}{S} > \frac{[\exp(\sum V_i \ln_i)^{1/s}]^k}{S} = Q^*. \end{aligned} \quad (\text{A.6})$$

The last expression determines the quota level for which reducing compression (increasing k) will have a positive effect on quota. Below Q^* increases in k will reduce Q_i . This limiting value has no simple analytical form, but can be calculated with the current data to be equal to 2.4486 so that (A.6) becomes $Q_i \gtrsim 2.45$.

- **Property 4:** *Changing compression changes the spread of the distribution but not the ranking of countries.* Ranking invariance is an important property since the purpose of compression

is to reduce concentration by transferring quota from the largest members to the smallest, but without creating the conflicts among benefited countries that could arise if their relative positions were altered. We can thus use (A.5) to examine the effect of an increase in k for two different countries i and j such that if

$$\begin{aligned} Q'_{ki} > Q'_{kj} &\implies V_i \ln_i S - \sum V_i \ln_i V_i > V_j \ln_j S - \sum V_i \ln_i V_j \\ &\frac{V_i}{S} (\ln_i S - \sum V_i \ln_i) > \frac{V_j}{S} (\ln_j S - \sum V_i \ln_i) \\ &Q_i (\ln_i S - \sum V_i \ln_i) > Q_j (\ln_j S - \sum V_i \ln_i) . \end{aligned} \quad (\text{A.7})$$

Note that if $Q_i > Q_j$ then $V_i > V_j$, and $\ln_i > \ln_j$ because the logarithmic function is monotonically increasing. In other words, as long as $Q_i > Q_j$ equation (A.7) will hold, and $Q'_{ki} > Q'_{kj}$. With Q^* being the limiting value for change in sign for Q'_k , this implies an increase in k will cause a proportionally larger outward movement for the countries furthest away from Q^* , and a resulting increase in the spread of the distribution. That is, a larger increase in the quota of i if both i and j are above Q^* , and a larger fall in the quota of j if both are below. The opposite is true for a reduction in k as it reverses the inequality ($Q'_{ki} < Q'_{kj}$) and compresses the distribution via greater movement towards the center for countries closer to Q^* .

Up to this point we have accounted for what happens to the distribution of quotas but not to its ranking, which is essential for the validity of this property. Especially in the case of a reduction in k where a larger inward movement for countries furthest away from the center of the distribution could potentially lead to a ranking change. To determine what happens to the ordering assume $Q_i > Q_j$, and let compression change to \tilde{k} such that $\tilde{V}_i = (\cdot)_i^{\tilde{k}}$ and $\tilde{S} = \sum_{i=1}^n \tilde{V}_i$. For the ranking to be preserved, after-the-change quotas \tilde{Q} must maintain their order; that is

$$\begin{aligned} \tilde{Q}_i = \tilde{V}_i / \tilde{S} &> \tilde{V}_j / \tilde{S} = \tilde{Q}_j \\ (\cdot)_i^{\tilde{k}} &> (\cdot)_j^{\tilde{k}} \\ (\cdot)_i &> (\cdot)_j , \end{aligned}$$

which must be the case since countries with larger quotas will have a larger linear combination of variables prior to compression. In other words, changing compression will alter the spread of the distribution but not the ranking of countries regardless of the direction of the change in k .

B. CHANGES IN FORMULA VARIABLES FOR EMDCs

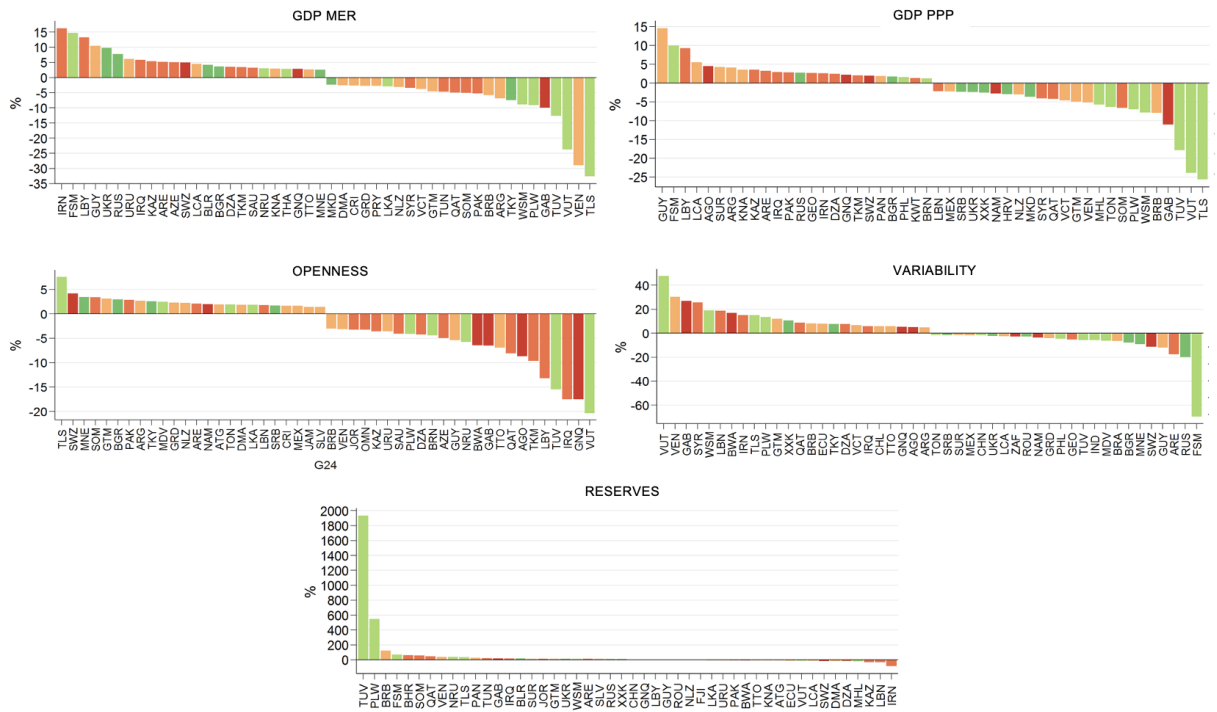


Figure 20: Largest changes by country in formula variables among EMDCs

C. KEY TO BOX PLOTS

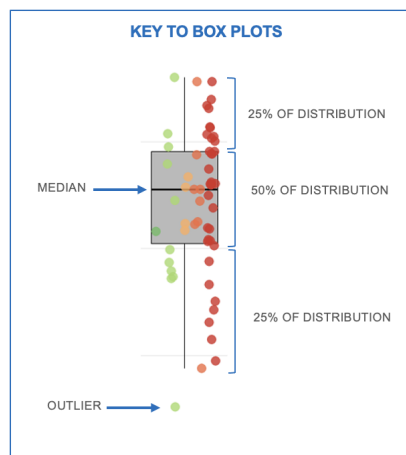


Figure 21: Key to reading the box plots used in the paper

D. CHAIR COMPOSITION

AUS_KOR	BENELUX	BRA	CAN_IR	CHN	CONG_SEN	EUR_TKY	FRA	GER	IND_SLK	IRAN_PK	ITA_GR
Australia Kiribati	Andorra Armenia	Brazil Cabo Verde Dominican Republic	Antigua and Barbuda Barbados	China	Benin Burkina Faso	Austria Belarus Czech Republic	France	Germany	Bangladesh Bhutan	Afghanistan Algeria	Albania Greece
Korea Marshall Islands Micronesia	Belgium Bosnia and Herzegovina Bulgaria	Ecuador Guyana	Belize Canada Dominica		Cameroon Central African Republic Chad	Hungary Kosovo Slovak Republic Slovenia			India Sri Lanka	Ghana Iran Libya	Italy Malta Portugal
Mongolia Nauru	Croatia Cyprus	Haiti Nicaragua	Grenada Ireland		Comoros Côte d'Ivoire Democratic Republic of the Congo Djibouti	Turkey				Morocco Pakistan	San Marino
New Zealand Palau Papua New Guinea	Georgia Israel	Panama Suriname	Jamaica St. Kitts and Nevis		Equatorial Guinea					Tunisia	
Samoa Seychelles Solomon Islands Tuvalu Vanuatu	Moldova Montenegro North Macedonia Romania The Netherlands Ukraine	Timor-Leste Trinidad and Tobago	St. Lucia St. Vincent and the Grenadines The Bahamas		Gabon Guinea Guinea-Bissau Madagascar Mali Mauritania Mauritius Niger Republic of Congo Rwanda São Tomé and Príncipe Senegal Togo						
JPN	NORD	OEDCE	RUS	SA_NIG	SAUDI	SE_ASIA	SOUTH_AM	SWI_POL	UAE_EGY	UK	USA
Japan	Denmark Estonia	Colombia Costa Rica	Russian Federation	Angola Botswana	Saudi Arabia	Brunei Darussalam Cambodia	Argentina Bolivia	Azerbaijan Kazakhstan Kyrgyz Republic Poland Serbia Switzerland Tajikistan Turkmenist an Uzbekistan	Bahrain Egypt Iraq Jordan Kuwait Lebanon Maldives Oman Qatar Syria United Arab Emirates Yemen	United Kingdom	United States
	Finland Iceland Latvia Lithuania Norway Sweden	El Salvador Guatemala Honduras Mexico Spain Venezuela		Burundi Eritrea Eswatini Ethiopia Kenya Lesotho Liberia Malawi Mozambique Namibia Nigeria Sierra Leone Somalia South Africa South Sudan Sudan Tanzania The Gambia Uganda Zambia Zimbabwe		Fiji Indonesia Lao P.D.R. Malaysia Myanmar Nepal Philippines Singapore Thailand Tonga Vietnam	Chile Paraguay Peru Uruguay				

Figure 22: Chair composition in alphabetical order

E. G-24 MEMBERS

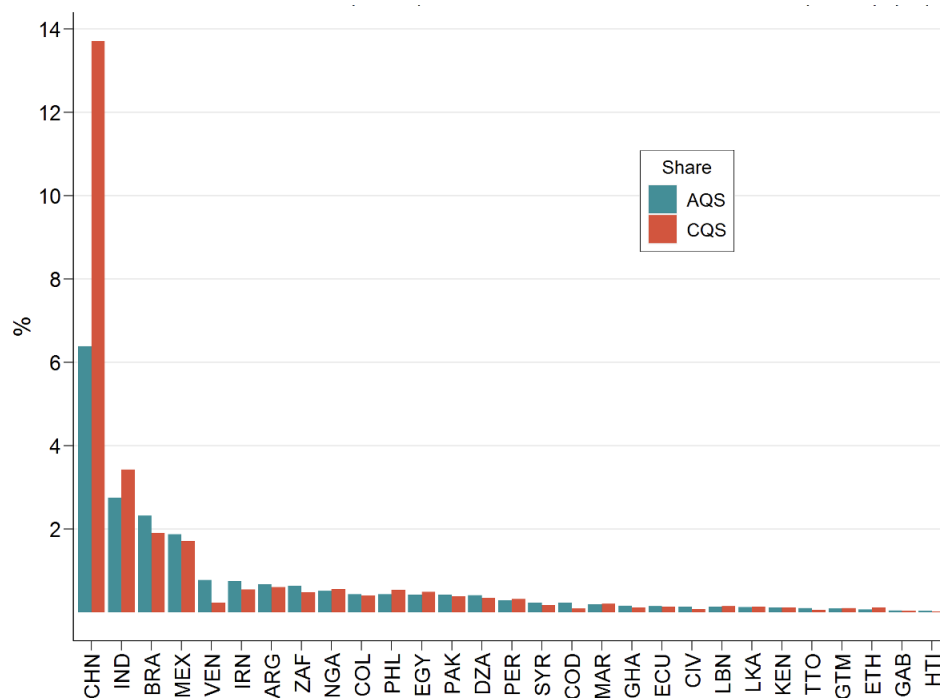


Figure 23: G-24 actual quota shares and calculated quota shares

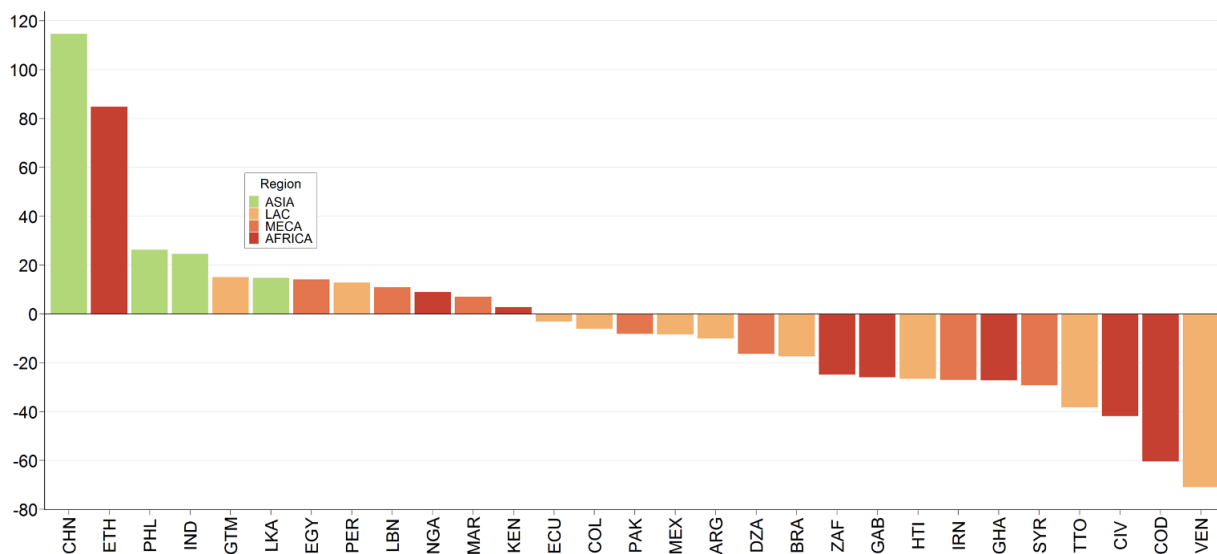


Figure 24: G-24 percentage change in actual quota shares and calculated quota shares

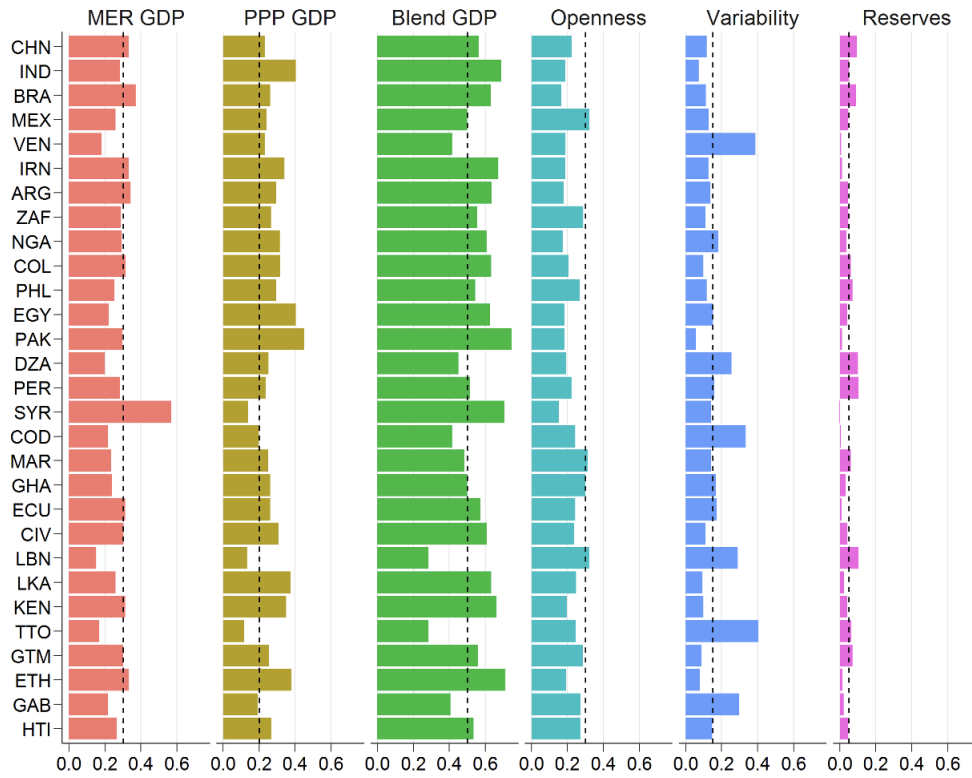


Figure 25: G-24 variable contributions prior to compression



Figure 26: G-24 contribution of GDP-blend variables

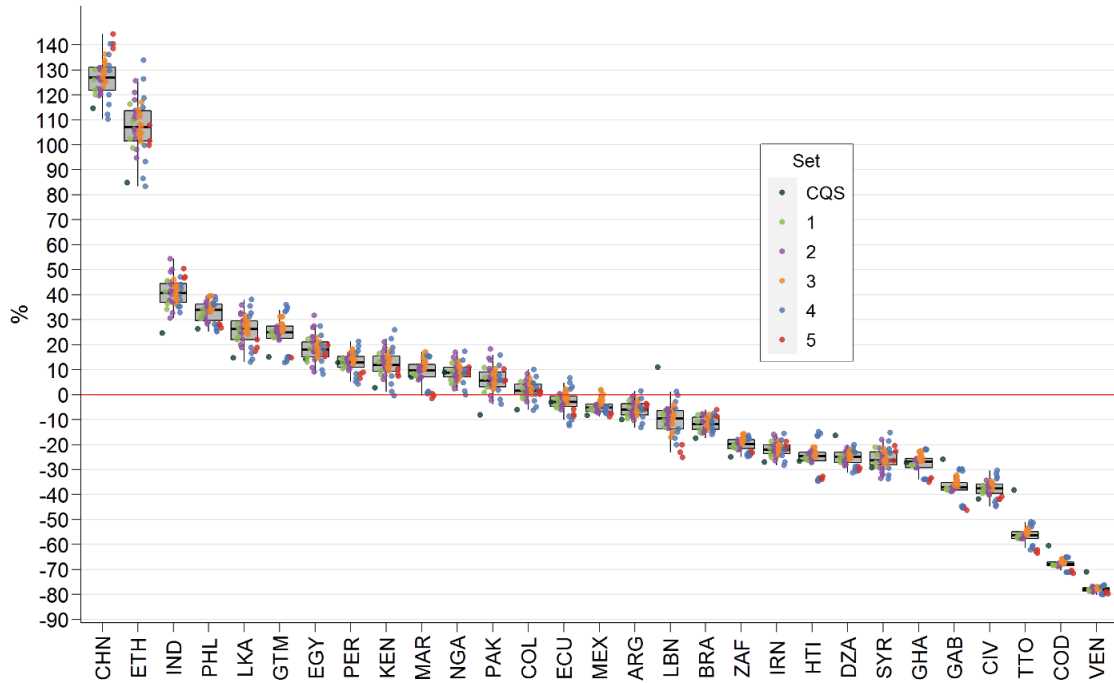


Figure 27: G-24 changes in Quota for all reform scenarios

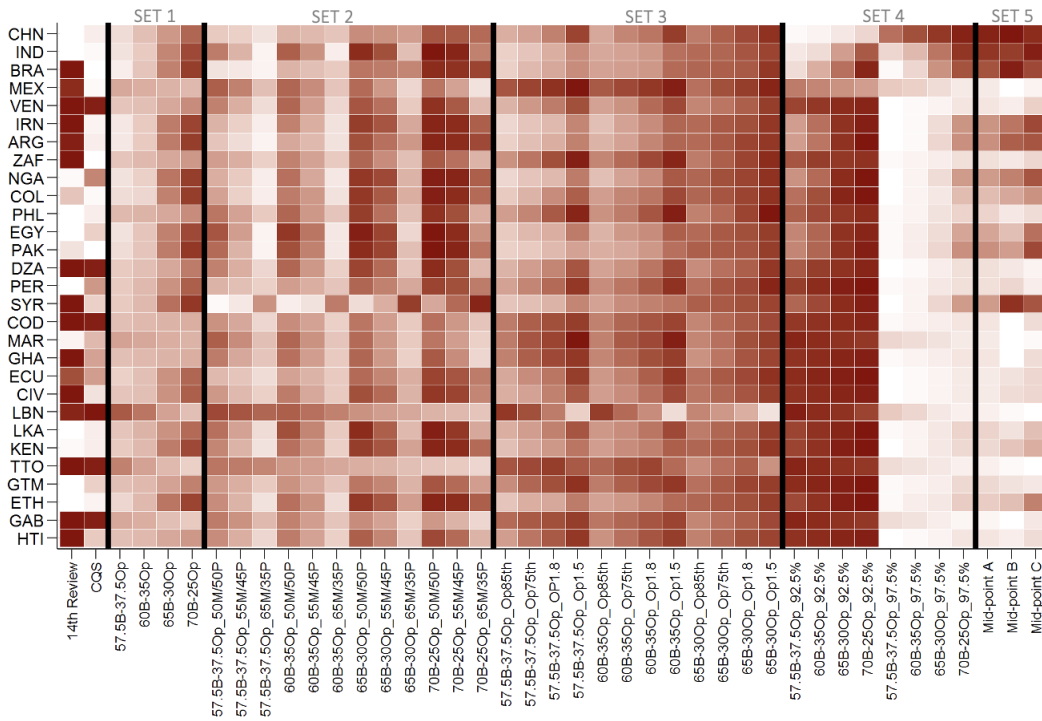


Figure 28: G-24 reform scenario ranking (darker=greater preference)

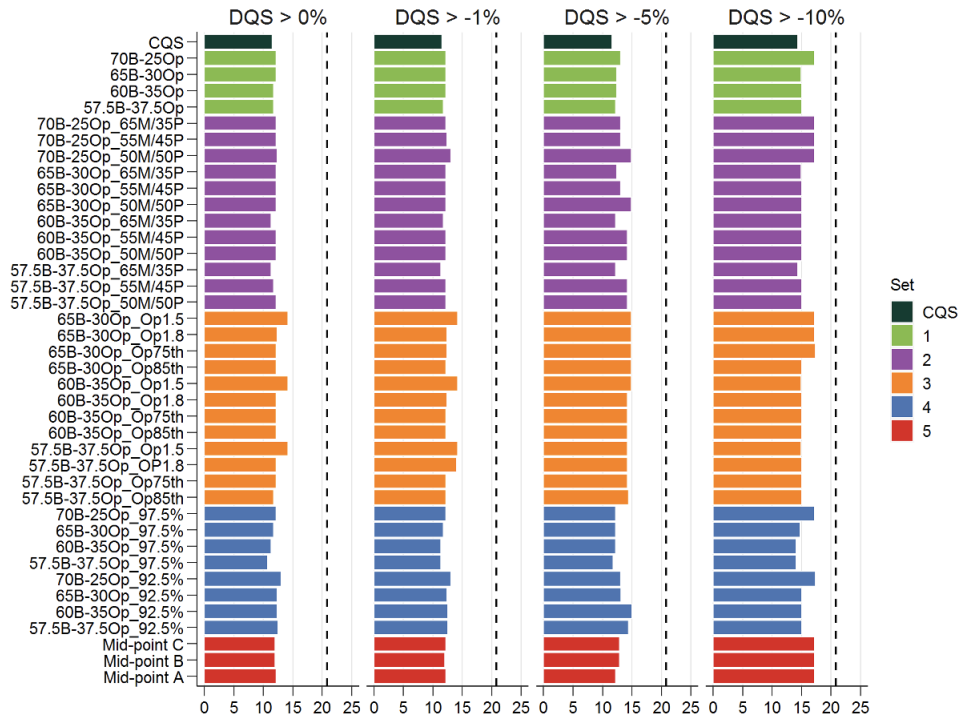


Figure 29: G-24 simulated votes by reform scenario

Country	14th Review	CQS	Variable with highest value	Variable with highest contribution	Preferred scenario
China	6.39	13.71	Reserves	Blend GDP	Mid-point B
India	2.75	3.43	Blend GDP	Blend GDP	70B-25Op_50M/50P
Brazil	2.31	1.91	Reserves	Blend GDP	14th Review
Mexico	1.87	1.71	Openness	Blend GDP	57.5B-37.5Op_1.5
Venezuela	0.78	0.23	Variability	Blend GDP	14th Review
Iran	0.75	0.55	Blend GDP	Blend GDP	14th Review
Argentina	0.67	0.60	Blend GDP	Blend GDP	70B-25Op_92.5%
South Africa	0.64	0.48	Blend GDP	Blend GDP	14th Review
Nigeria	0.51	0.56	Variability	Blend GDP	70B-25Op_92.5%
Colombia	0.43	0.40	Blend GDP	Blend GDP	70B-25Op_92.5%
Philippines	0.43	0.54	Reserves	Blend GDP	65B-30Op_1.5
Egypt	0.43	0.49	Blend GDP	Blend GDP	70B-25Op_50M/50P
Pakistan	0.43	0.39	Blend GDP	Blend GDP	70B-25Op_50M/50P
Algeria	0.41	0.34	Reserves	Blend GDP	14th Review
Peru	0.28	0.32	Reserves	Blend GDP	70B-25Op_92.5%
Syria	0.23	0.16	MER GDP	Blend GDP	14th Review
Democratic Republic of the Congo	0.22	0.09	Variability	Blend GDP	14th Review
Morocco	0.19	0.20	Reserves	Blend GDP	57.5B-37.5Op_1.5
Ghana	0.15	0.11	Variability	Blend GDP	14th Review
Ecuador	0.15	0.14	Variability	Blend GDP	70B-25Op_92.5%
Côte d'Ivoire	0.14	0.08	Blend GDP	Blend GDP	14th Review
Lebanon	0.13	0.15	Reserves	Openness	CQS
Sri Lanka	0.12	0.14	Blend GDP	Blend GDP	70B-25Op_92.5%
Kenya	0.11	0.12	Blend GDP	Blend GDP	70B-25Op_92.5%
Trinidad and Tobago	0.10	0.06	Variability	Variability	14th Review
Guatemala	0.09	0.10	Reserves	Blend GDP	70B-25Op_92.5%
Ethiopia	0.06	0.12	Blend GDP	Blend GDP	70B-25Op_92.5%
Gabon	0.05	0.03	Variability	Blend GDP	14th Review
Haiti	0.03	0.03	Blend GDP	Blend GDP	14th Review

Figure 30: G-24 preferences summary (only scenarios preferred by more than 3 countries are highlighted)

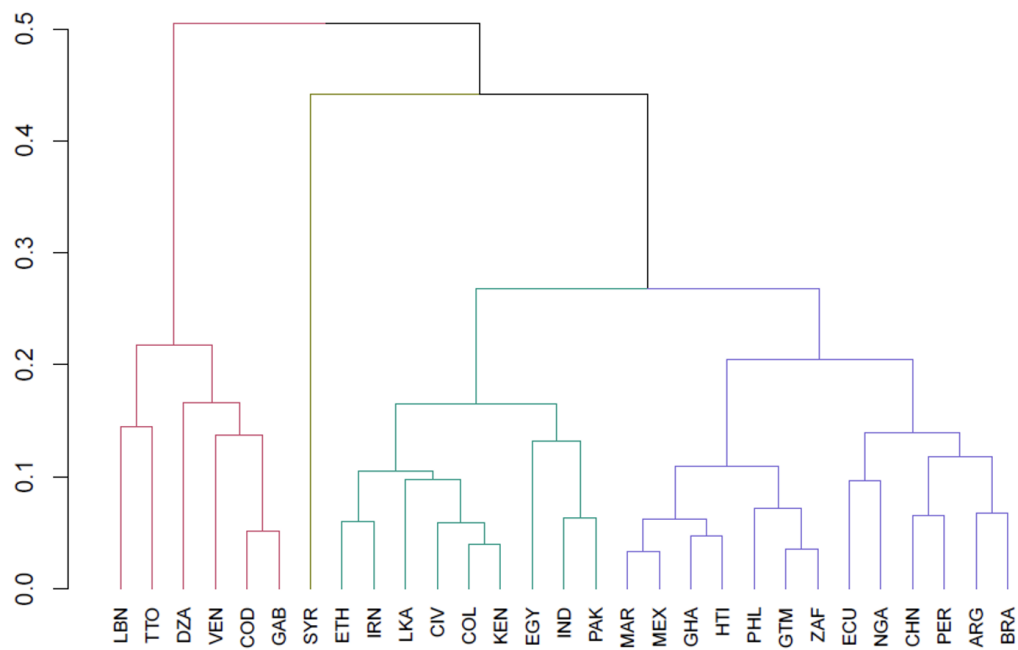


Figure 31: G24 dendrogram for variable contributions ($k = 4$)